

LECTURE NOTES

ON

FLUID MECHANICS

PREPARED BY

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(DEPT. OF MECHANICAL ENGG.)

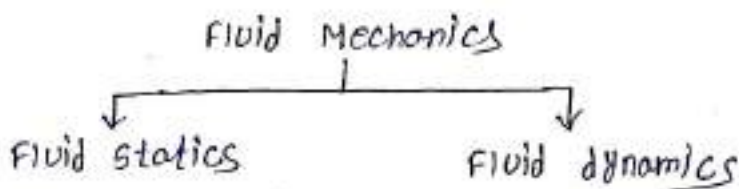
GOVERNMENT POLYTECHNIC, PURI

properties of fluid

Fluid:- It is such kind of matter which can easily flow from one place to another place.

Fluid Mechanics:-

It is the branch of science which deals with the effect of forces on fluid.

properties of fluids① Mass and volume relation

① Density or Mass density (ρ) = $\frac{m}{V}$

SI unit - kg/m^3

② Specific volume (v) = V/m

SI unit - m^3/kg

• Relation between ρ and v

$$\Rightarrow v = \frac{1}{\rho} \quad (\text{Reciprocal})$$

③ Specific weight or weight density (w) = $\frac{W}{V}$

SI unit = N/m^3

• Relation between ρ and w

$$\Rightarrow w = \frac{\text{weight}}{\text{Volume}} = \frac{\text{Mass} \times \text{acceleration due to gravity}}{\text{Volume}}$$

$$= \frac{mg}{V} = \left(\frac{m}{V}\right) \times g = \rho g.$$

$$\Rightarrow w = \rho g.$$

$$\textcircled{4} \text{ specific gravity (s)} = \frac{\text{density of fluid under study}}{\text{density of a standard fluid}}$$

density

soil - 800 kg/m^3

water - 1000 kg/m^3

air -

where, $M = \text{mass}$

$V = \text{volume}$

$v = \text{specific volume}$

$\rho = \text{density or mass density}$

$W = \text{weight}$

$w = \text{specific weight}$

$g = \text{acceleration due to gravity}$

$s = \text{specific gravity}$

① If the weight density of a liquid is 8.1 N/m^3 , calculate its density, specific volume and specific gravity.

Ans: Given data,

$$w = 8.1 \text{ N/m}^3$$

$$\text{density } (\rho) = \frac{w}{g} = \frac{8.1}{9.81} = 0.82 \text{ kg/m}^3$$

$$\text{specific volume } (v) = \frac{1}{\rho} = \frac{1}{0.82} = 1.21 \text{ m}^3/\text{kg}$$

$$\text{specific gravity } (s) = \frac{0.82}{1000} = 8.2 \times 10^{-4}$$

① If a fluid is having a specific gravity of 0.85 then calculate its density, specific weight and specific volume.

Ans: Given data,

$$s = 0.85$$

$$\text{we know } s = \frac{\rho}{1000}$$

$$\begin{aligned} \text{density } (\rho) &= s \times 1000 \\ &= 0.85 \times 1000 = 850 \text{ kg/m}^3 \end{aligned}$$

$$\begin{aligned} \text{specific weight } (\omega) &= \rho g \\ &= 850 \times 9.81 = 8338.5 \text{ N/m}^3 \end{aligned}$$

$$\begin{aligned} \text{specific volume } (v) &= \frac{V}{m} \text{ or } \frac{1}{\rho} \\ &= \frac{1}{850} = 1.176 \times 10^{-3} \text{ m}^3/\text{kg} \end{aligned}$$

② An oil is having a weight of 8 kN for 5 m³ of the volume. Calculate the density, weight density, specific volume and specific gravity of the oil.

Ans: Given data,

$$\text{weight } (W) = 8 \text{ kN} = 8000 \text{ N}$$

$$\text{volume } (V) = 5 \text{ m}^3$$

$$\begin{aligned} \text{weight density } (\omega) &= \frac{W}{V} \\ &= \frac{8000}{5} = 1600 \text{ N/m}^3 \end{aligned}$$

$$\begin{aligned} \text{density } (\rho) &= \frac{\omega}{g} \\ &= \frac{1600}{9.81} = 163.09 \text{ kg/m}^3 \end{aligned}$$

$$\text{specific volume} = \frac{1}{\rho} = \frac{1}{163.09} = 6.131 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$\text{Specific gravity (s)} = \frac{\rho}{1000}$$

$$= \frac{163.09}{1000} = 0.16309.$$

viscosity

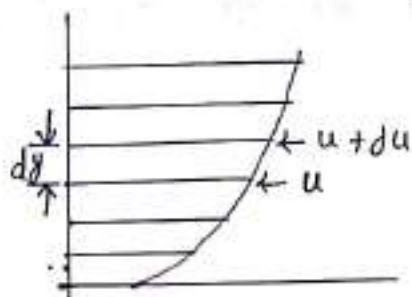
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viscosity is defined as the property of the fluid which offers resistance to the movement of one layer of fluid to that of the adjacent layer above it.

→ More viscous - movement will be slow.

Newton's law of viscosity

The law states that the shear stress on a fluid element layer is directly proportional to the rate of change of velocity between adjacent layers (or shear strain). The proportionality constant is known as the coefficient of viscosity.



We know, stress & strain

in fluid case, shear stress & $\frac{du}{dy}$

$$\Rightarrow \tau = \mu \frac{du}{dy}$$

This proportionality constant μ is also known as dynamic viscosity.

The CGS unit of dynamic viscosity is known as poise.

$$\Rightarrow \mu = \frac{\tau dy}{du}$$

where, τ = shear stress

μ = co-efficient of viscosity or dyne viscosity or viscosity

$\frac{du}{dy}$ = velocity gradient or strain.

dy = distance between the layers

units of viscosity

SI unit - Pa·s

CGS unit - poise

$\tau = N/m^2$ or Pa

$du = m/s$

$dy = m$

$$\frac{du}{dy} = \frac{m/s}{m} = 1/s$$

$$\therefore \mu = \frac{\tau}{du/dy} = \frac{Pa}{1/s} = Pa \cdot s$$

⊕ Relation between poise and SI unit of viscosity

SI unit

$$Pa \cdot s = N/m^2 \cdot s$$

CGS unit

$$poise = \frac{dyne}{cm^2} \cdot s$$

$$1 Pa \cdot s = \frac{1 N}{1 m^2} \cdot 1 s$$

$$= \frac{10^5 dyne}{(100 cm)^2} \cdot 1 s = \frac{10^5 dyne}{10^4 cm^2} \cdot s = \frac{10 dyne}{cm^2} \cdot s = 10 poise$$

$$1 Pa \cdot s = 10 poise$$

① A plate 0.025 mm distant from a fixed plate moves at 50 m/s and requires a force of 1.471 N for every 1 m² area to maintain the speed. Determine the fluid viscosity between the plates in poise.

Ans: Given data,

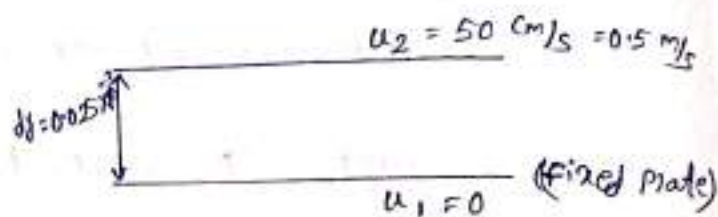
$$dy = 0.025 \text{ mm} = 0.025 \times 10^{-3} \text{ m}$$

$$F = 1.471 \text{ N}$$

$$A = 1 \text{ m}^2$$

$$u_2 = 50 \text{ m/s}$$

$$\mu = ?$$



We know, shear stress (τ) = F/A

$$= \frac{1.471 \text{ N}}{1 \text{ m}^2} = 1.471 \text{ N/m}^2$$

$$\therefore \tau = \mu \frac{du}{dy}$$

$$\Rightarrow \mu = \frac{\tau \times dy}{du}$$

$$= \frac{1.471 \times 0.025 \times 10^{-3}}{0.5}$$

$$= 7.355 \times 10^{-5} \text{ Pa}\cdot\text{s}$$

$$\mu \text{ (in poise)} = 7.355 \times 10^{-5} \text{ Pa}\cdot\text{s}$$

$$= 7.355 \times 10^{-5} \times 10$$

$$= 7.355 \times 10^{-4} \text{ poise (Ans)}$$

② Two plates are placed at a distance of 0.15 mm apart. The lower plate is fixed while the upper plate is pulled at a velocity of 0.3 m/s. Calculate the shear stress if the viscosity is 1.5 poise.

Ans: Given data

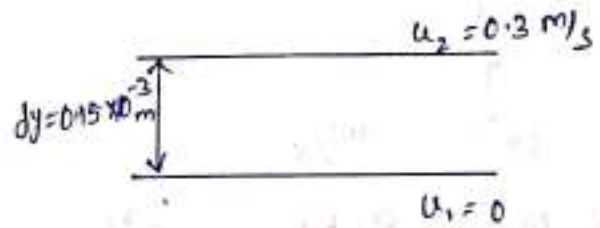
$$dy = 0.15 \text{ mm} = 0.15 \times 10^{-3} \text{ m}$$

$$u_1 = 0$$

$$u_2 = 0.3 \text{ m/s}$$

$$\tau = ?$$

$$\mu = 1.5 \text{ poise} = 0.15 \text{ Pa}\cdot\text{s}$$



We know, $1 \text{ Pa}\cdot\text{s} = 10 \text{ poise}$

$$1 \text{ poise} = 0.1 \text{ Pa}\cdot\text{s}$$

$$1.5 \text{ poise} = 0.1 \times 1.5 \text{ Pa}\cdot\text{s} \\ = 0.15 \text{ Pa}\cdot\text{s}$$

$$\text{Change in velocity } (du) = u_2 - u_1$$

$$= 0.3 - 0$$

$$= 0.3 \text{ m/s}$$

$$\therefore \text{shear stress } (\tau) = \mu \frac{du}{dy}$$

$$= 0.15 \times \frac{0.3}{0.15 \times 10^{-3}}$$

$$= \frac{0.3}{10^{-3}}$$

$$= 0.3 \times 10^3 = 300 \text{ N/m}^2 \quad (\text{Ans})$$

kinematic viscosity

It is the ratio between the dynamic viscosity and density of the fluid.

$$\nu = \frac{\mu}{\rho}$$

$$\text{SI unit} = \text{m}^2/\text{s}$$

$$\text{CGS unit} = \text{stoke} = \text{cm}^2/\text{s}$$

Relation between stoke and SI unit of kinematic viscosity

$$\begin{aligned} \nu &= \frac{\mu}{\rho} = \frac{\text{Pa} \cdot \text{s}}{\text{kg}/\text{m}^3} \\ &= \frac{\text{Pa} \cdot \text{s} \cdot \text{m}^3}{\text{kg}} \\ &= \frac{\text{N} \cdot \text{s} \cdot \text{m}^3}{\text{m}^2 \cdot \text{kg}} \\ &= \frac{\text{N} \cdot \text{s} \cdot \text{m}}{\text{kg}} \\ &= \left(\frac{\text{N}}{\text{kg}} \right) \cdot \text{s} \cdot \text{m} \\ &= \left(\text{m}/\text{s}^2 \right) \cdot \text{s} \cdot \text{m} \\ &= \text{m}^2/\text{s} \\ &= 10^4 \text{ stoke} \end{aligned}$$

$$\therefore F = MA$$

$$F/m = a$$

$$\text{N}/\text{kg} = \text{a}$$

$$\text{N}/\text{kg} = \text{m}/\text{s}^2$$

① In a stream of glycerine the velocity gradient at a point is 0.25 m/s per unit 1 m . The density of the fluid is 1268.4 kg/m^3 . Calculate the shear stress at that point if the kinematic viscosity is $6.3 \times 10^{-4} \text{ m}^2/\text{s}$.

Ans: Given data,

$$du = 0.25 \text{ m/s}$$

$$dy = 1 \text{ m}$$

$$\rho = 1268.4 \text{ kg/m}^3$$

$$\tau = \mu \frac{du}{dy} = ?$$

$$\nu = \frac{\mu}{\rho} = 6.3 \times 10^{-4} \text{ m}^2/\text{s}$$

We know, $\nu = \frac{\mu}{\rho}$

$$\Rightarrow 6.3 \times 10^{-4} = \frac{\mu}{1268.4}$$

$$\Rightarrow \mu = 6.3 \times 10^{-4} \times 1268.4$$

$$\Rightarrow \mu = 0.79$$

$$\therefore \text{shear stress } (\tau) = \mu \frac{du}{dy}$$

$$= 0.79 \times \frac{0.25}{1}$$

$$= 0.1975 \text{ Pa. (Ans)}$$

② Find the kinematic viscosity of an oil having density 980 kg/m^3 . When at a certain point the shear stress is 0.25 N/m^2 and velocity gradient is $0.3/\text{s}$.

Ans given data,

$$\gamma = \frac{\mu}{\rho} = ?$$

$$\rho = 980 \text{ kg/m}^3$$

$$\tau = 0.25 \text{ N/m}^2$$

$$du = 0.3$$

$$dy = 1$$

$$\text{We know } \Rightarrow \tau = \mu \frac{du}{dy}$$

$$\Rightarrow 0.25 = \mu \times \frac{0.3}{1}$$

$$\Rightarrow \mu = \frac{0.25}{0.3}$$

$$\Rightarrow \mu = 0.83$$

$$\therefore \gamma = \frac{\mu}{\rho}$$

$$= \frac{0.83}{980} = 8.46 \times 10^{-4} \text{ m}^2/\text{s}$$

$$= 8.46 \times 10^{-4} \times 10^4$$

$$= 8.46 \text{ stokes}$$

③ Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 2.

Ans. given data,

$$\mu = ?$$

$$\nu = 6 \text{ stokes} = 6 \times 10^{-4} \text{ m}^2/\text{s}$$

$$S = 2$$

we know, $s = \frac{f_{\text{liquid}}}{f_{\text{H}_2\text{O}}} = \frac{f_{\text{liquid}}}{1000}$

$\Rightarrow 2 = \frac{f_{\text{liquid}}}{1000}$

$\Rightarrow f_{\text{liquid}} = 2000$

$\therefore \eta = \frac{\ell l}{f}$

$\Rightarrow \ell l = \eta \times f$

$= 6 \times 10^{-4} \times 2000$

$= 1.2 \text{ pas}$

*) Difference between solid and fluid.

Solid	Fluid
i) definite shape and definite volume.	i) NO fixed shape and no fixed volume, it assumes containers shape.
ii) intermolecular force of attraction is more.	ii) intermolecular force of attraction is less.
iii) solid can not flow easily from one place to another place.	iii) fluid can easily flow from one place to another place.
iv) A solid can resist a deformation force while at rest.	iv) Fluid can not resist deformation force, it moves or flows under the action of force.
v) Ex- wood, glass	v) Ex- oil, water

$\frac{61}{8} = 7.625$

Surface tension on a liquid jet :-

diameter = d

length = L

Force due to surface tension = $\sigma \times 2L$

pressure force = $P \times \text{area of semi jet}$
 $= P \times L \times d$

$$\therefore P \times L \times d = \sigma \times 2L$$

$$\Rightarrow \sigma = \frac{P \times L \times d}{2L} = \frac{P \times d}{2}$$

① The surface tension of water in contact with air is 0.145 N/m . The pressure inside a droplet of water is 0.02 N/cm^2 greater than the outside pressure calculate the diameter of the droplet of water.

Ans: Given data,

$$\sigma = 0.145 \text{ N/m}$$

$$P = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \text{ N/m}^2$$

$d = ?$

$$\text{we know, } \sigma = \frac{P d}{4}$$

$$\Rightarrow d = \frac{\sigma \times 4}{P}$$

$$\Rightarrow d = \frac{0.145 \times 4}{0.02 \times 10^4} = 2.9 \times 10^{-3} \text{ m}$$

NOTE

$$\frac{\text{N}}{\text{cm}^2} = \frac{\text{N}}{\text{cm} \times \text{cm}} = \frac{\text{N}}{10^{-2} \text{ m} \times 10^{-2} \text{ m}} = \frac{\text{N}}{10^{-4} \text{ m}^2} = 10^4 \frac{\text{N}}{\text{m}^2}$$

② The surface tension of water in contact with air is 0.0716 N/m . The pressure difference between the outside air and inside water is 0.0147 N/cm^2 . Calculate the diameter of the droplet.

Ans: Given data,

$$\sigma = 0.0716 \text{ N/m}$$

$$P = 0.0147 \text{ N/cm}^2 = 0.0147 \times 10^4 \text{ N/m}^2$$

$$d = ?$$

$$\text{We know, } \sigma = \frac{Pd}{4}$$

$$\Rightarrow d = \frac{\sigma \times 4}{P}$$

$$\Rightarrow d = \frac{0.0716 \times 4}{0.0147 \times 10^4} = 1.94 \times 10^{-3} \text{ m.}$$

③ Find the surface tension in a soap bubble of 30mm diameter. When the inside pressure is 1.962 N/m^2 above the atmosphere.

Ans: Given data,

$$\sigma = ?$$

$$d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$$

$$P = 1.962 \text{ N/m}^2$$

$$\sigma = \frac{Pd}{8}$$

$$= \frac{1.962 \times 30 \times 10^{-3}}{8} = 7.35 \times 10^{-3} \text{ N/m.}$$

20.03.23

capillarity

capillarity is defined as the rise or fall of a liquid in a very small tube when it is held vertically in the liquid.

It is denoted by 'h'.

$$h = \frac{4\sigma \cos \theta}{\rho g d}$$

unit = m, mm.

where, h = height rise or fall

σ = surface tension

ρ = density of the liquid

g = acceleration due to gravity

θ = angle of contact between the surface of glass tube and the liquid. Generally it is taken as 0° for water.

① calculate the capillary rise in a glass tube of 1.25 mm diameter when immersed vertically in water, take the surface tension as 0.145 N/m .

Ans given data,

$$h = ?$$

$$d = 1.25 \text{ mm} = 1.25 \times 10^{-3} \text{ m}$$

$$\sigma = 0.145 \text{ N/m}$$

$$\text{capillarity (h)} = \frac{4\sigma \cos \theta}{\rho g d}$$

$$= \frac{4 \times 0.145 \times \cos 0^\circ}{1000 \times 9.81 \times 1.25 \times 10^{-3}} = 0.04 \text{ m.}$$

② calculate the capillarity in a glass tube of $\phi = 2.5 \text{ mm}$ diameter held vertically in mercury having a surface tension of 0.52 N/m . Take the specific gravity of mercury as 13.6 and angle of contact as 130° .

Ans: Given data,

$$d = 2.5 \text{ mm}$$

$$h = ?$$

$$\sigma = 0.52 \text{ N/m}$$

$$s_m = 13.6$$

$$\theta = 130^\circ$$

We know,

$$\text{specific gravity} = \frac{\text{density under study}}{\text{density on standard fluid}}$$

$$\Rightarrow S_m = \frac{\rho_m}{1000}$$

$$\Rightarrow 13.6 = \frac{\rho_m}{1000}$$

$$\Rightarrow \rho_m = 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$$\therefore h = \frac{4\sigma \cos \theta}{\rho g d}$$

$$= \frac{4 \times 0.52 \times \cos 130^\circ}{13600 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= -4.00 \times 10^{-3} \text{ m}$$

UNIT - 2

FLUID PRESSURE AND ITS MEASUREMENTS

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad \left[\text{SI unit} - \text{N/m}^2 \text{ or Pa} \right]$$

- * Measurement device -
- Manometer
 - Bourdon tube
 - Pressure gauge

$$* \text{ pressure head } (P) = h \rho g$$

where, h = height of liquid column

ρ = density of the liquid

g = gravity

$$\begin{aligned} 1 \text{ bar} &= 10^5 \text{ Pa} \\ 1 \text{ kPa} &= 10^3 \text{ Pa} \end{aligned}$$

④ Pressure :-

If the force is uniformly distributed over a cross-sectional area 'A' then the pressure is defined as the ratio of force to that of the pressure of area.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A}$$

unit - N/m^2

* Intensity pressure :-

We always take the force to be acting perpendicular to the cross-sectional area. If we take (small force) 'dF' acting on the area 'dA' in the normal direction then the ratio is known as intensity of pressure or simply pressure.

$$P = \frac{dF}{dA}, \text{ SI unit} = \text{N/m}^2$$

* pressure head :-

According to the microstatic law be kind and defined the pressure head as :-

$$h = \frac{P}{\rho g}, \text{ SI unit} = \text{m}$$

It is the vertical depth of any point from the free surface

① convert 10 mm of mercury to pascal unit.

Ans: 10 mm of Hg = 10^{-2} m of Hg

$$\begin{aligned} \Rightarrow P &= h \rho g \\ &= 10^{-2} \times 13600 \times 9.81 \\ &= 1334.16 \text{ Pa} \end{aligned}$$

$$\left[\begin{array}{l} \because 10 \text{ mm Hg} = 10^{-2} \text{ m} \\ \rho_{\text{mercury}} = 13600 \end{array} \right]$$

② The pressure intensity at a point in the fluid is given 4.9 N/cm^2 . Find a corresponding height of the fluid when it is (a) water

(b) oil of specific gravity 0.8

Ans: Given data,

$$P = 4.9 \text{ N/cm}^2 = 4.9 \times 10^4 \text{ N/m}^2$$

(a) water

$$h = \frac{P}{\rho g} = \frac{4.9 \times 10^4}{1000 \times 9.81} = 4.99 \text{ m}$$

(b) oil of specific gravity 0.8

$$h = \frac{P}{\rho g} = \frac{4.9 \times 10^4}{\rho_{oil} \times 9.81}$$
$$= \frac{4.9 \times 10^4}{800 \times 9.81}$$
$$= 6.24 \text{ m.}$$

$$\therefore S = \frac{\rho_{fluid}}{\rho_{standard}}$$
$$\Rightarrow 0.8 = \frac{\rho_{oil}}{1000}$$
$$\Rightarrow \rho_{oil} = 0.8 \times 1000$$
$$= 800$$

③ Calculate the pressure due to a column of 0.4m of
(a) water
(b) oil of specific gravity 0.9
(c) mercury ($\rho = 13600$)

Ans: Given data,

$$h = 0.4 \text{ m}$$

(a) water

$$P = h \rho g$$
$$= 0.4 \times 1000 \times 9.81$$
$$= 3924 \text{ N/m}^2$$

(b) oil of specific gravity 0.9

$$P = h \rho g$$
$$= 0.4 \times 900 \times 9.81$$
$$= 3531.6$$

$$\therefore S = \frac{\rho_{oil}}{\rho_{standard}}$$
$$\Rightarrow 0.9 = \frac{\rho_{oil}}{\rho_{standard}}$$

(c) mercury

$$P = h \rho g$$
$$= 0.4 \times 13600 \times 9.81$$
$$= 53366.4$$

$$\Rightarrow \rho_{oil} = 0.9 \times 1000$$
$$= 900$$

④ Find the corresponding height of water at a point which has a pressure of 20m of oil. Take the specific gravity of oil as 0.8.

Ans: Given data,

$$h_{oil} = 20m$$

$$S = 0.8, \rho_{oil} = 800 \text{ kg/m}^3$$

$$P = h_{oil} \times \rho_{oil} \times g$$

$$= h_{water} \times \rho_{water} \times g$$

$$\therefore \Rightarrow h_{oil} \times \rho_{oil} \times g = h_{water} \times \rho_{water} \times g$$

$$\Rightarrow h_{water} = \frac{h_{oil} \times \rho_{oil} \times g}{\rho_{water} \times g}$$

$$= \frac{20 \times 800}{1000} = 16m.$$

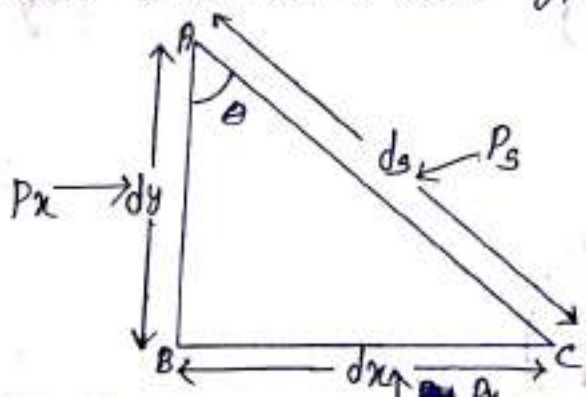
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Pascal's law :-

statement - It states that the pressure or intensity of pressure at a point in a static fluid is equal in all direction.

proof

Let us take a small fluid element of "wedge shaped" whose dimensions are given in the diagram.



P_x , P_s and P_y are the pressure elements acting on the surface AB, AC, BC respectively. The width of the surface is taken as 1 unit and the weight of the fluid element is neglected ($\cong 0$) as it is very small.

Force on AB \rightarrow

$$P_x \times \text{Area of AB} \\ = P_x \times (dy \times 1) = P_x \cdot dy$$

Force on BC \rightarrow

$$P_y \times (\text{Area of BC}) \\ = P_y \times (dx \times 1) = P_y \cdot dx$$

Force on AC \rightarrow

$$P_s \times \text{Area of AC} \\ = P_s \times (ds \times 1) = P_s \cdot ds$$

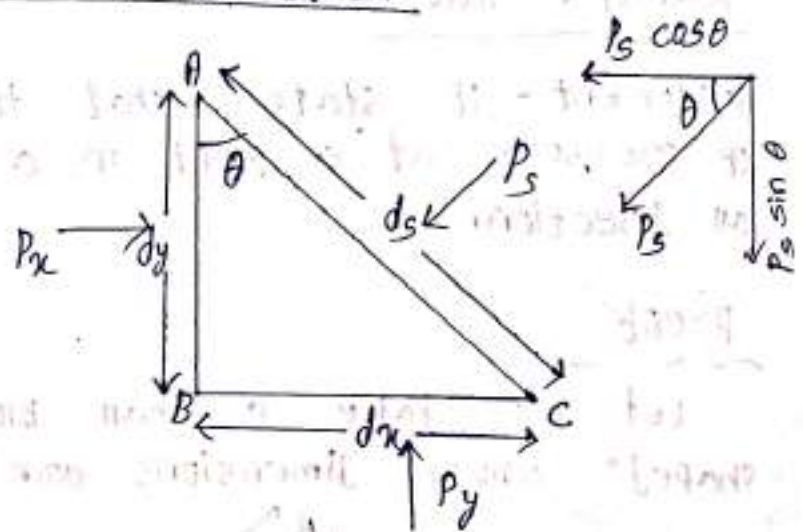
Force balance in x and y direction

x-direction \rightarrow

$$P_x \cdot dy = P_s ds \cos \theta \\ \Rightarrow P_x \cdot dy = P_s ds \left(\frac{dy}{ds} \right) \\ \Rightarrow P_x \cdot dy = P_s \cdot dy \\ \Rightarrow P_x = P_s \quad \text{--- (i)}$$

y-direction \rightarrow

$$P_y \cdot dx = P_s ds \sin \theta \\ \Rightarrow P_y \cdot dx = P_s ds \left(\frac{dx}{ds} \right)$$



$$\Rightarrow P_y \cdot dx = P_s \cdot dx$$

$$\Rightarrow P_y = P_s \quad \text{----- (ii)}$$

Combining eqn (i) and (ii), we can write

$$\boxed{P_x = P_y = P_s} \quad (\text{proved})$$

5.4.2023

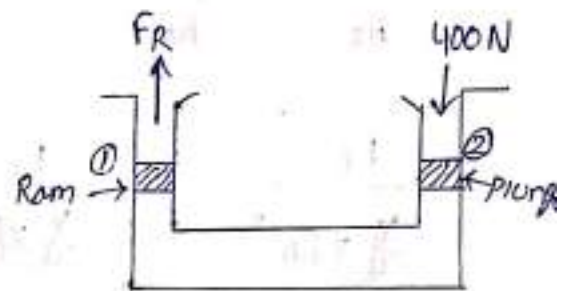
- ① A hydraulic press has a ram (d_r) of 30 cm diameter and plunger of (d_p) = 5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N.

Ans: Given data,

$$\text{Ram} \rightarrow d_r = 30 \text{ cm} = 0.3 \text{ m}$$

$$\text{plunger} \rightarrow d_p = 5 \text{ cm} = 0.05 \text{ m}$$

$$\text{Force} \rightarrow F_p = 400 \text{ N}$$



$$P_1 = P_2$$

$$\Rightarrow \frac{F_R}{A_R} = \frac{F_p}{A_p}$$

$$\Rightarrow \frac{F_R}{\frac{\pi}{4} \times (d_r)^2} = \frac{400}{\frac{\pi}{4} \times (d_p)^2}$$

$$\Rightarrow F_R = 400 \times \left(\frac{d_r}{d_p} \right)^2$$

$$= 400 \times \left(\frac{0.3}{0.05} \right)^2$$

$$= 400 \times 36$$

$$= 14400 \text{ N}$$

② A hydraulic press has a ram of 20 cm diameter and a plunger of 4 cm diameter. It is used for lifting a weight of 20 kN. Find the force required at the plunger.

Ans: Given data,

$$\text{Ram} \rightarrow d_r = 20 \text{ cm} = 0.2 \text{ m}$$

$$\text{Plunger} \rightarrow d_p = 4 \text{ cm} = 0.04 \text{ m}$$

$$\text{Weight (W)} = 20 \text{ kN} = 20000 \text{ N} \cdot (F_R)$$

$$F_R = ?$$

$$\therefore P_1 = P_2$$

$$\Rightarrow \frac{F_R}{A_R} = \frac{F_P}{A_P}$$

$$\Rightarrow \frac{F_R}{\frac{\pi}{4} \times (d_1)^2} = \frac{F_P}{\frac{\pi}{4} \times (d_2)^2}$$

$$\Rightarrow \frac{20 \times 10^3}{(0.2)^2} = \frac{F_P}{(0.04)^2}$$

$$\Rightarrow F_P = \frac{20 \times 10^3}{(0.2)^2} \times (0.04)^2$$

$$\Rightarrow F_P = 800 \text{ N}$$

* Pressure measurement

06.04.23

Absolute pressure :-

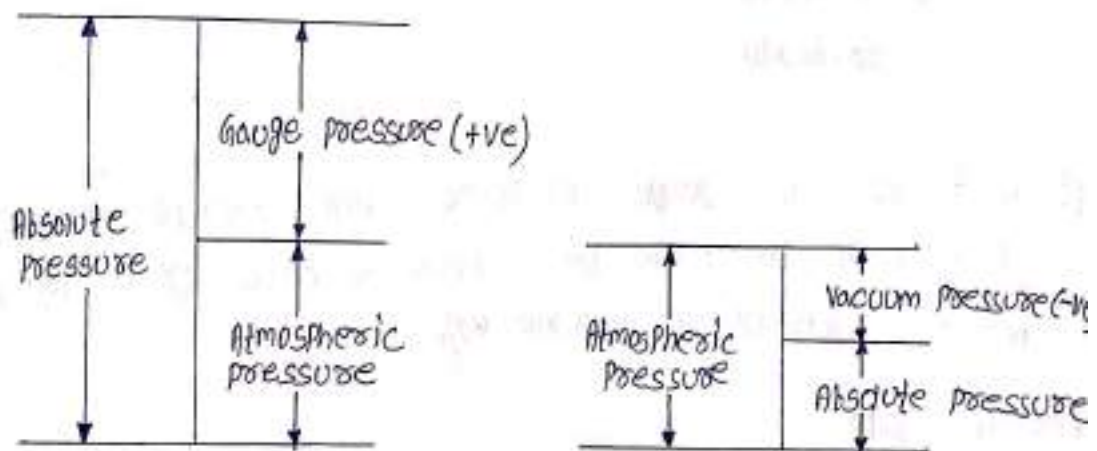
Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure. (0 reading).

Gauge pressure :-

Gauge pressure is defined as the pressure reading in the pressure measuring instrument where the atmospheric pressure is taken as reference.

Vacuum pressure :-

It is also known as negative gauge pressure, which gives a reading below the atmospheric pressure.



→ The standard value of atmospheric pressure is taken as 1.01325 bar or 760 mm of Hg at sea level.

→ The pressure measured from the absolute zero pressure is called absolute pressure.

Absolute pressure = Atmospheric pressure + Gauge pressure

$$P_{abs} = P_{atm} + P_g \quad (P_{abs} > P_{atm})$$

Absolute pressure = Atmospheric pressure - Vacuum pressure

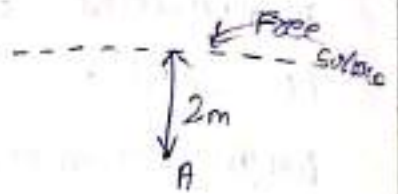
$$P_{abs} = P_{atm} - P_{vac} \quad (P_{abs} < P_{atm})$$

① determine the gauge and absolute pressure at a point which is 2m below the free surface of water. Take the atmospheric pressure as 10.1043 N/cm^2

Ans: Given data,

$$P_{\text{atm}} = 10.1043 \text{ N/cm}^2 = 10.1043 \times 10^4 \text{ N/m}^2$$

$$h_A = 2 \text{ m}$$



$$\begin{aligned} P_{\text{gauge}} &= h \rho g \\ &= 2 \times 10^3 \times 9.81 \\ &= 19.62 \times 10^3 \text{ Pa} \end{aligned}$$

$$\begin{aligned} P_{\text{abs}} &= P_{\text{atm}} + P_{\text{gauge}} \\ &= 10.1043 \times 10^4 + 19.62 \times 10^3 \\ &= 120663 \text{ Pa} \\ &= 120.66 \text{ kPa} \end{aligned}$$

② What are the gauge pressure and absolute pressure at a point 3m below the free surface of a liquid having a density of $1.53 \times 10^3 \text{ kg/m}^3$.

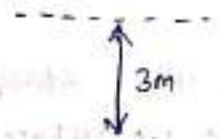
Ans: Given data,

$$P_{\text{gauge}} = ?$$

$$P_{\text{abs}} = ?$$

$$h_A = 3 \text{ m}$$

$$\rho = 1.53 \times 10^3 \text{ kg/m}^3$$



$$\begin{aligned} P_{\text{gauge}} &= h \rho g \\ &= 3 \times 1.53 \times 10^3 \times 9.81 \\ &= 45027.9 \text{ Pa} \\ &= 45.0279 \text{ kPa} \end{aligned}$$

$$\begin{aligned}
 P_{\text{obs}} &= P_{\text{atm}} + P_{\text{gauge}} \\
 &= 101325 + 45027.9 \\
 &= 146352.9 \text{ Pa} \\
 &= 146.3529 \text{ kPa}
 \end{aligned}$$

③ Convert 26 cm of mercury vacuum pressure in to absolute pressure.

Ans: Given data,

$$h_g = 26 \text{ cm} = 0.26 \text{ m}$$

$$\begin{aligned}
 P &= h_g \rho \\
 &= 0.26 \times 13600 \times 9.81 \\
 &= 34688.16 \text{ Pa} \\
 &= 34.688 \text{ kPa}
 \end{aligned}$$

$$\begin{aligned}
 P_{\text{abs}} &= P_{\text{atm}} - \text{vacuum pressure} \\
 &= 101325 - 34688.16 \\
 &= 66636.84 \text{ Pa} \\
 &= 66.636 \text{ kPa}
 \end{aligned}$$

13.04.23

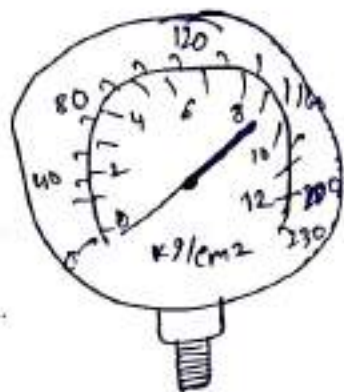
④ pressure measuring instruments

The fluid pressure can be measured by using following measuring devices.

(i) Bourdon tube pressure gauge

(ii) Manometers $\left\{ \begin{array}{l} \rightarrow \text{simple manometer} \\ \rightarrow \text{differential manometer} \end{array} \right.$

⊕ Bourdon tube pressure gauge :-



- The most common type of pressure gauge is a Bourdon tube pressure gauge.
- It is generally used for measuring high pressure.
- A Bourdon gauge uses a coiled tube, which as it expands due to pressure increase causes a rotation of an arm connected to the tube.
- It consists of a hollow coiled metallic tube usually made of bronze or nickel. One end of the tube is sealed and other end is connected to the pipe whose pressure is to be measured.
- When the pressure in the hollow tube increases, the tube will tend to uncoil and when the pressure decreases it will tend to coil more tightly.
- This movement is transferred through a rack and pinion arrangement connected to a pointer over a calibrated dial, directly giving the pressure of fluid. This gauge is capable of measuring both positive and ~~negative gauge pressure~~.

⊕ Manometers :-

Manometers are defined as the devices used for measuring the pressure at a point. They are classified as :-

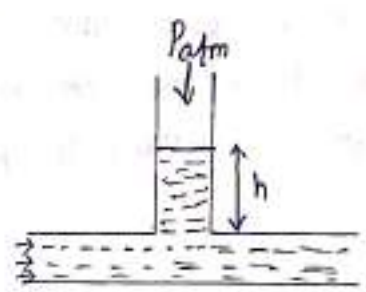
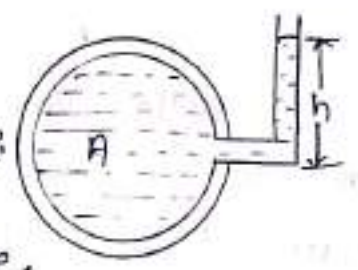
- ① simple manometers
- ② differential manometers

Simple manometers :-

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere as shown in figure.

piezometer

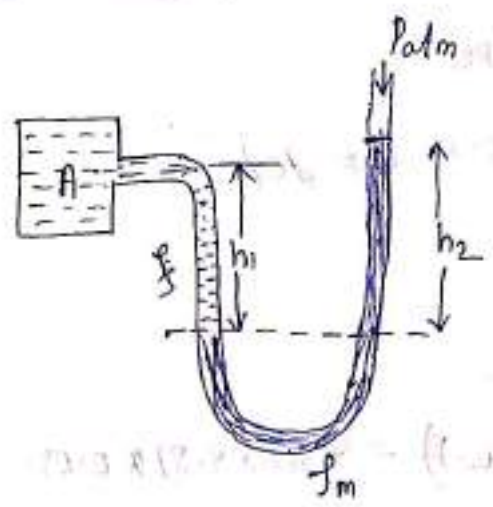
It is a simple type of manometer generally used for measuring the gauge pressure. Its a simple single tube connected to the system and is generally open to atmosphere.



$P_{sys} > P_{atm}$

$P_{abs} = P_{atm} + \rho \cdot h$ height of the liquid
 $= P_{atm} + \rho g h$

Simple u-tube manometer



$P_{sys} > P_{atm}$

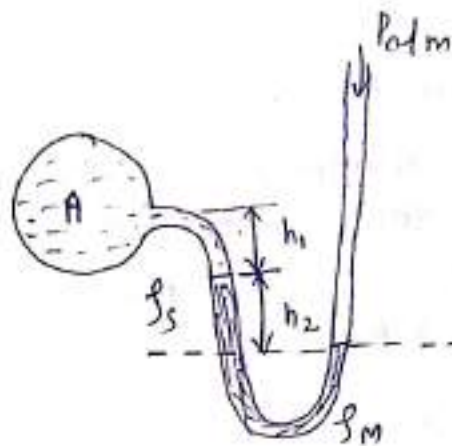
$P_{abs} = ?$

Left	Right
$P_A + h_1 \rho_s g$	$= P_{atm} + h_2 \rho_m g$
$P_A = P_{atm} + h_2 \rho_m g - h_1 \rho_s g$	

$$P_{\text{sys}} < P_{\text{atm}}$$

$$P_A + h_1 \rho_s g + h_2 \rho_m g = P_{\text{atm}}$$

$$P_A = P_{\text{atm}} - h_1 \rho_s g - h_2 \rho_m g$$



17.04.23

- ① The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which fluid of specific gravity 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of the fluid in the pipe if difference of mercury level in the two limbs is 20 cm.

Ans! Given data,

$$S_{\text{fluid}} = 0.9$$

$$\rho_{\text{fluid}} = 900 \text{ kg/m}^3$$

$$h_2 = 20 \text{ cm} = 0.2 \text{ m}$$

$$h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$$

Let P_A = pressure of fluid in pipe

Equating the pressure above x-x, we get

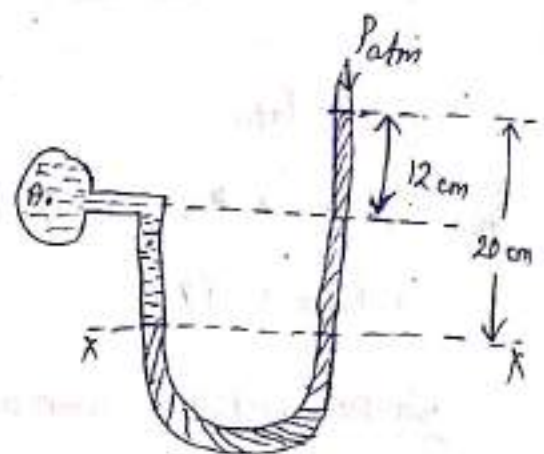
$$P_A + \rho_f g h_1 = P_{\text{atm}} + \rho_m g h_2$$

$$\Rightarrow P_A = (P_{\text{atm}} + \rho_m g h_2) - \rho_f g h_1$$

$$= (101325 + (13600 \times 9.81 \times 0.2)) - (900 \times 9.81 \times 0.08)$$

$$= 127301.88 \text{ Pa}$$

$$= 12.7301 \times 10^4 \text{ Pa}$$



$$\begin{aligned}
 P_{\text{gauge}} &= P_{\text{A}} - P_{\text{atm}} \\
 &= 127301.88 - 101325 \\
 &= 25976.88 \\
 &= 2.5976 \times 10^4 \text{ Pa.}
 \end{aligned}$$

OR

$$\begin{aligned}
 P_{\text{gauge}} &= (13600 \times 9.81 \times 0.2) - (900 \times 9.81 \times 0.08) \\
 &= 266832 - 70632 \\
 &= 25976.88 \text{ Pa} \\
 &= 2.5976 \times 10^4 \text{ Pa.}
 \end{aligned}$$

- ② A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of specific gravity 0.8 having a vacuum pressure is blowing. The other end of the manometer is open to atmosphere. The difference of the mercury level in the two limbs is 40 cm and height of the fluid in the left side from the centre of the pipe is 15 cm below. calculate the gauge reading and absolute reading.

Ans: Given data,

$$S_F = 0.8$$

$$\rho_F = 800 \text{ kg/m}^3$$

$$h_2 = 40 \text{ cm} = 0.4 \text{ m}$$

$$h_1 = 15 \text{ cm} = 0.15 \text{ m}$$

$$P_A = ?$$

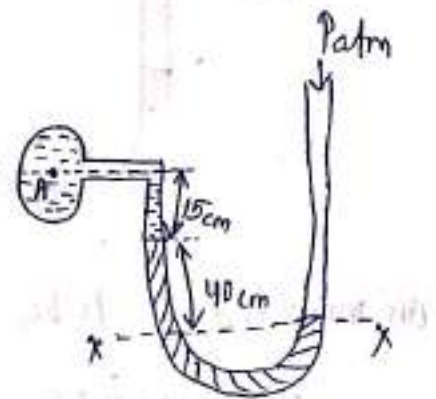
$$P_{\text{gauge}} = ?$$

Equating both side pressure above $x-x$, we get

$$P_A + \rho_F g h_1 + \rho_M g h_2 = P_{\text{atm}}$$

$$P_A = P_{\text{atm}} - \rho_F g h_1 - \rho_M g h_2$$

$$\begin{aligned}
 &= 101325 - (800 \times 9.81 \times 0.15) - (13600 \times 9.81 \times 0.4) \\
 &= 46781.4 \text{ Pa}
 \end{aligned}$$



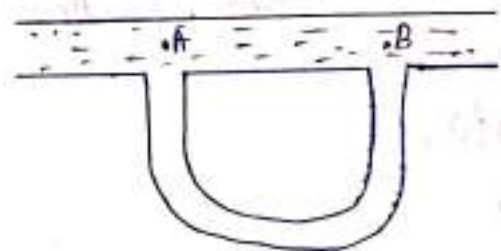
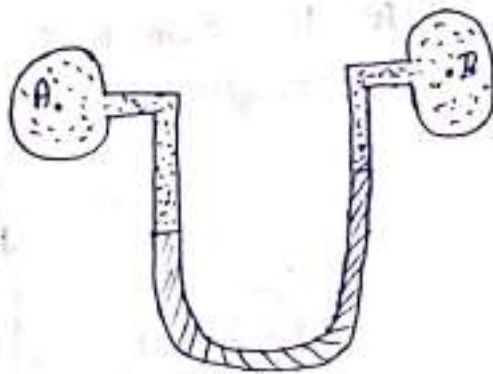
$$\begin{aligned}
 P_{\text{gauge}} &= - \rho_f g h_1 - \rho_m g h_2 \\
 &= - 800 \times 9.81 \times 0.15 - 13600 \times 9.81 \times 0.4 \\
 &= - 54543.6 \\
 &= - 5.4543 \times 10^4 \text{ Pa.}
 \end{aligned}$$

18.04.23

Differential Manometers

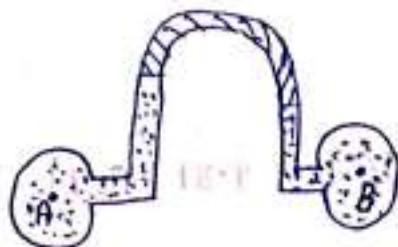
(i) U-tube differential manometer

Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid whose two ends are connected to the points, whose difference of pressure is to be measured.



(ii) Inverted U-tube differential manometer

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressure.



① Find the pressure difference of two system A and B. If the difference in the mercury level of the manometer is 18 cm and system A contains a fluid which specific gravity 1.5 and system B contains a fluid of specific gravity 0.9 as shown in figure.

Ans: Given data,

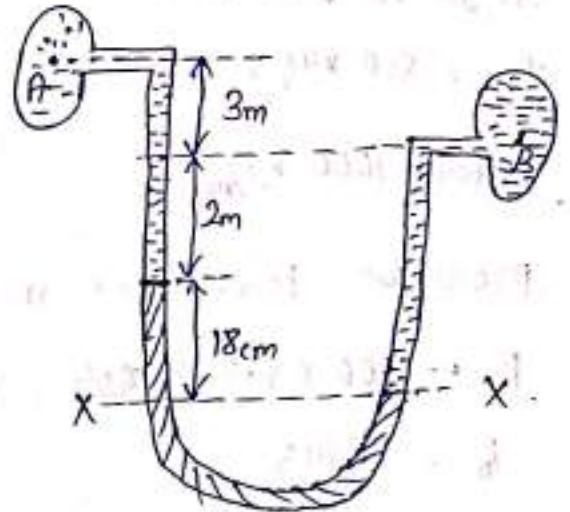
pressure difference of Hg = 18 cm = 0.18 m

SP. gr of A = 1.5

$\rho_A = 1500 \text{ kg/m}^3$

SP. gr of B = 0.9

$\rho_B = 900 \text{ kg/m}^3$



pressure above x-x in system A

$$P_A + (1500 \times 9.81 \times 5) + (13600 \times 9.81 \times 0.18)$$

$$P_A + 97589.88 \text{ Pa}$$

pressure above x-x in system B

$$P_B + 900 \times 2.18 \times 9.81$$

$$P_B + 19247.22 \text{ Pa}$$

pressure difference = Equating two pressure

$$\Rightarrow P_A + 97589.88 = P_B + 19247.22$$

$$\Rightarrow P_A - P_B = 97589.88 - 19247.22$$

$$= 78342.66$$

$$= 7.8342 \times 10^4 \text{ Pa}$$

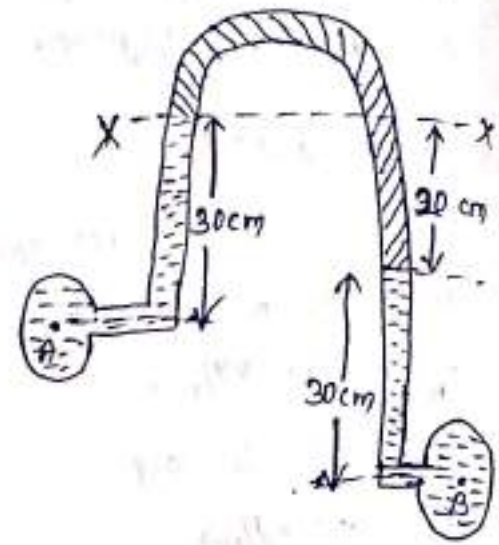
② As shown in figure an inverted differential manometer contains an oil of specific gravity 0.8. It is connected to two pipes A and B which contains water in it. Find the pressure difference between A and B.

Ans: Given data,

$$\text{SP. gr. of oil} = 0.8$$

$$\rho_{\text{oil}} = 800 \text{ kg/m}^3$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$



pressure below X-X in system A

$$P_A = 1000 \times 9.81 \times 0.3$$

$$P_A = 2943$$

pressure below X-X in system B

$$P_B = (1000 \times 9.81 \times 0.3) + (800 \times 9.81 \times 0.2)$$

$$P_B = 2943 + 1569.6$$

$$P_B = 4512.6$$

$$P_B = 4512.6$$

pressure difference,

= equating two pressure

$$\Rightarrow P_A - 2943 = P_B - 4512.6$$

$$\Rightarrow P_B - P_A =$$

$$= 4512.6 - 2943$$

$$= 1569.6 \text{ N/m}^2$$

15.05.23

CHAPTER - 3

HYDROSTATICS

Hydrostatic pressure:-

Total hydrostatic pressure is defined as the force exerted by static fluid on a surface either plane or curve when the fluid comes in contact with the surface. This force always acts normal to the surface. It is a static property where the relative motion between adjacent layers is zero (velocity gradient = 0). This is also known as total pressure.

Centre of pressure:-

It is a point where the total pressure is assumed to be acting on the surface.

Total pressure and centre of pressure on immersed body :-

(i) vertical body :-

Consider a plane vertical surface where 'A' is the total area of the surface, 'G' is the centre of gravity of plane surface, point P centre of pressure, 'h' distance of centre of gravity of the area from free surface of liquid.

h^* - distance of centre of pressure from free surface of liquid.

Total pressure (F) :-

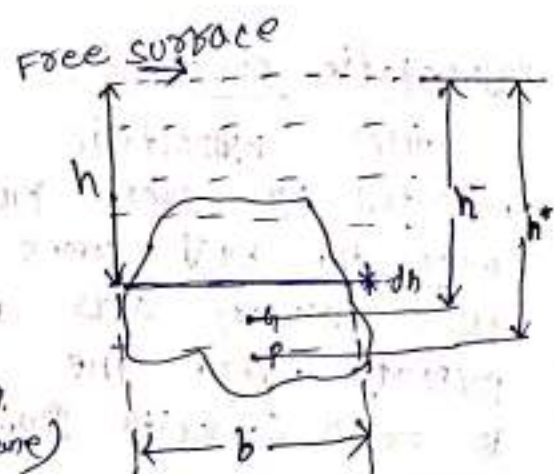
This can be calculated by dividing the entire surface into a no. of small parallel strips and then integrating the force acting on a small strip over the entire surface. so we can write total pressure force on the entire surface is 'F'.

$$F = \int dF = \int p \times dA$$

$$= \int \rho g h \times dA$$

$$= \rho g \int h \times dA$$

$$= \rho g A h \quad (\text{equal bar horizontal and vertical bar})$$



(i) A rectangular plane surface lies in vertical plane in water having a width of 2m and depth of 3m. calculate the total pressure on the plane surface when the upper edge is horizontal and coincide with water surface.

(ii) 2.5m below the free surface.

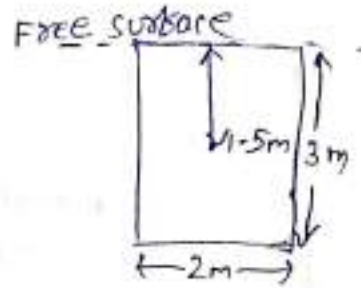
Ans: Given data,

$$\rho = 1000 \text{ kg/m}^3$$

$$A = 2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$$

$$\bar{h} = 1.5 \text{ m}$$

$$g = 9.81$$



(i) coincide with water surface

$$\text{Total pressure (F)} = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 6 \times 1.5$$

$$= 88290 \text{ N}$$

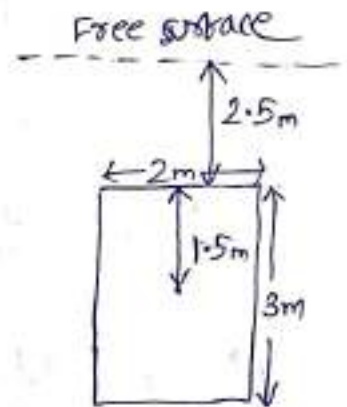
(ii) 2.5m below the free surface

$$\bar{h} = 2.5 + 1.5 = 4 \text{ m}$$

$$\text{Total force (F)} = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 6 \times 4$$

$$= 235440 \text{ N}$$



② determine the total pressure on a circular plate of diameter 1.5m which is placed vertically in water in such a way that the centre of the plate is 2m below the free surface of water.

Ans: $d = 1.5 \text{ m}$

$$\bar{h} = 2 \text{ m}$$

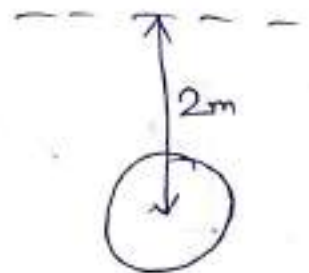
$$\text{Area} = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} \times (1.5)^2 = 1.76 \text{ m}^2$$

$$\text{Total force (F)} = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 1.76 \times 2$$

$$= 34531.2 \text{ N}$$



center of pressure :- (vertical)

It is calculated by the principle of moments when we can equate the moment of the resultant hydrostatic force to that of the sum of the moments of the components about the same axis. so we can write

$$\begin{aligned}
 F \times h^* &= \int dF \times h \\
 &= \int p \times dA \times h \\
 &= \int \rho g h \times dA \times h \\
 &= \rho g \int h^2 dA
 \end{aligned}$$

$$F h^* = \rho g I_0^*$$

$$\begin{aligned}
 h^* &= \frac{\rho g I_0}{F} \\
 &= \frac{\cancel{\rho g} I_0}{\cancel{\rho g} A \bar{h}} \\
 &= \frac{I_0}{A \bar{h}} \\
 &= \frac{I_G + A \bar{h}^2}{A \bar{h}}
 \end{aligned}$$

$$h^* = \frac{I_G}{A \bar{h}} + \bar{h}$$

I_0 = ^{Area} Moment of inertia of the body about the free surface.
 I_G = Moment of inertia of the area about its centre of gravity.

* Buoyancy Force

- When a ~~bed~~ solid object is wholly or partly immersed in a fluid, the fluid molecules are continuously striking the submerged surface of the object.
- The forces due to these impacts can be combined into a single force called buoyant force which counteracts the weight.
- This ~~tendency~~ ^{tend to} buoyant force lift the body up. This tendency for an immersed body to be lifted up in the fluid, due to upward force, is known as buoyancy.

$$\text{Buoyancy force } (F_b) = \rho g V$$

where ρ = density of object

g = Acceleration due to gravity.

V = volume of object.

Ex - while trying to push the ball in water, you can feel difficult to push the ball because of upward pressure of fluid.

* ARCHIMEDES PRINCIPLE

Archimedes principle says when an object immersed in a fluid there is upward force acting on it, this upward force is called buoyancy force.

→ Buoyancy force is equal to the weight of fluid displaced by the object.

Force of buoyancy = weight of fluid displaced.

Ex → when a object immersed in fluid, it loses its weight. The loss of weight of object is equal to upward buoyancy force acting on it.

~~Ex~~

Loss of weight of object = Buoyancy force = weight of fluid displaced.

$$\Rightarrow W_a - W_f = F_B$$

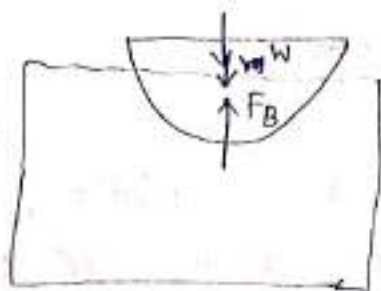
W_a - Normal weight in air
 W_f = Apparent weight of object when it is immersed in fluid.

Application

A ship floats on water.
Hot air balloon, hydrometers.

(*) META CENTRE OF BUOYANCY

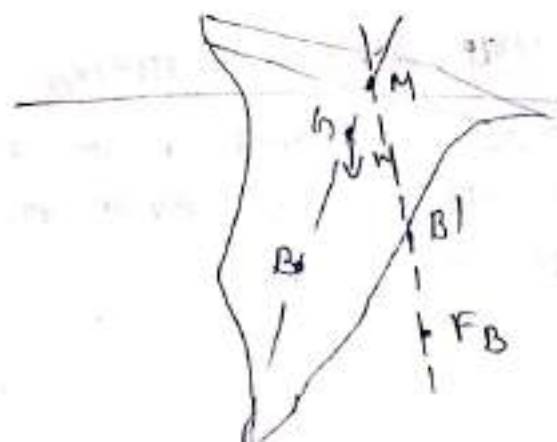
It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of fluid displaced.



center of gravity
center of buoyancy

(*) META CENTRE

It is defined as the point about which a body starts oscillating when the body tilted by a small angle. The meta centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.



→ The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M . This point M is called meta centre.

⊙ META CENTRIC HEIGHT:-

The distance MG , i.e. the distance between the meta centre of a floating body and the centre of gravity of the body is called meta centric height.

⊙ FLOATATION

Floatation can be defined as the tendency of an object to rise up to the upper levels of fluid or to stay on the surface of the fluid.

The opposite of floatation is sinking and can be defined as the tendency of an object to go to the lower levels of the fluid.

UNIT-4KINEMATICS OF FLOWTYPES OF FLUID FLOW(i) Steady and unsteady flow :-

→ steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc at a point do not change with time.

→ unsteady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc. at a point or particular section change with time.

(ii) Uniform and nonuniform flow :-

→ uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space. (i.e. length or direction of flow)

→ Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space.

(iii) Rotational and irrotational flow :-

→ If the fluid particles rotate about their own axis while flowing along the stream line, then the flow is known as rotational flow.

→ If the fluid particle does not rotate about their own axis while flowing along the streamline then the flow is known as irrotational flow.

(iv) Laminar and turbulent flow :-

→ If the fluid particles move along well-defined paths also known as stream line, which are parallel to each other then the flow is known as laminar flow.

→ If the fluid particles moves on zigzag path or does not move stream line i.e called turbulent flow.

Reynold's number :-

This constant helps us to determine whether the flow is laminar or turbulent.

$$R = \frac{VD}{\nu}$$

where, V = Mean velocity of fluid flow

D = Diameter of pipe

ν = kinematic viscosity

If the Reynold's no. value is less than 2000 ($R < 2000$) the flow is known as laminar. If it is greater than 4000 ($R > 4000$) then the flow is turbulent.

(v) compressive and incompressible flow :-

ρ = constant \rightarrow incompressible (ex- water)

does not constant \rightarrow compressive

(vi) 1D, 2D, 3D flow

⊛ Energy conservation

Bernoulli's equation :-

It states that in a steady, ideal flow of an incompressible fluid the total energy (pressure energy, kinetic energy and potential energy or datum energy) at any point is constant.

24-03-23

Bernoulli's equation

①

Mathematically, $\frac{p}{w} + \frac{v^2}{2g} + z = \text{constant}$

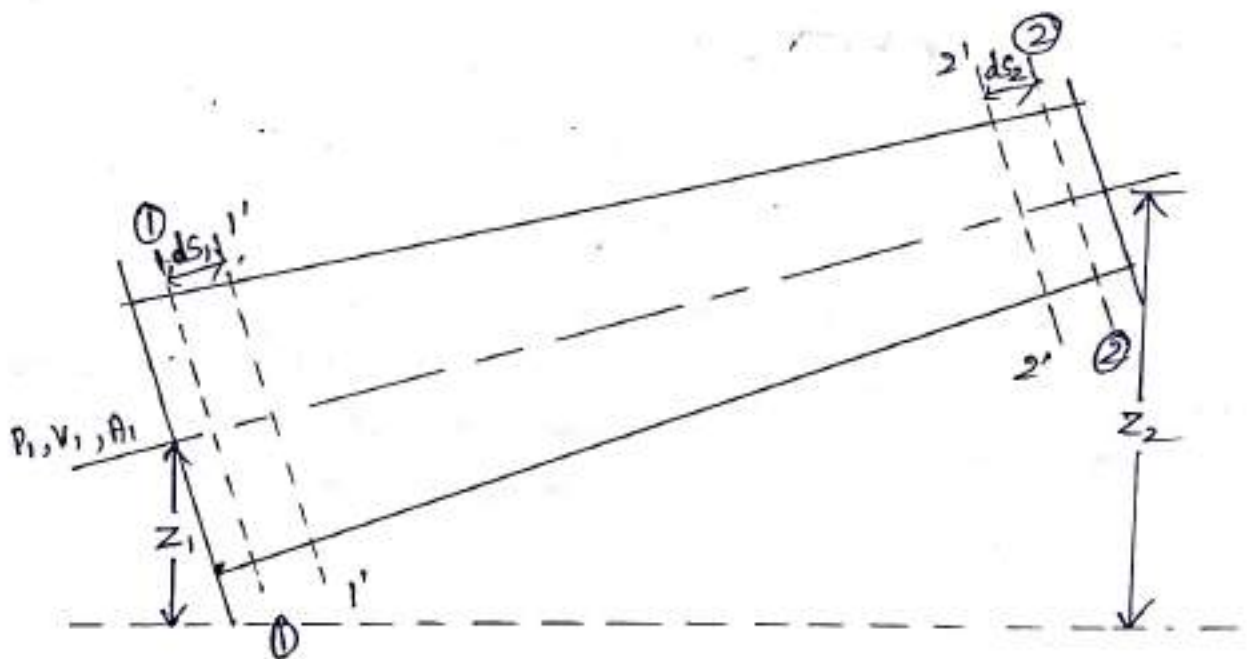
where, $\frac{p}{w} = \text{pressure energy}$

$\frac{v^2}{2g} = \text{kinetic energy}$

$z = \text{datum energy}$

Proof

- ↳ considers steady flow of compressible liquid through a non-uniform pipe lying entirely in the x-y plane.
- ↳ Let us consider two sections 1-1 and 2-2 in an uniformly varying pipe as shown in figure 1.



Let, $p_1 = \text{pressure at 1-1}$

$v_2 = \text{velocity at 1-1} = v_1$

$z_1 = \text{height of 1-1 above datum}$

$A_1 = \text{area of pipe at 1-1}$

p_2, v_2, z_2 and A_2 are corresponding values at 2-2.

Let us consider two sections 1-1 and 2-2 in an unsteady varying pipe as shown in figure.

The movement of fluid ~~is~~ between 1-1 and 2-2 equivalent to the movement of the liquid between ~~1-1 and 2-2~~ 1-1 and 1'-1' to 2-2 and 2'-2' the remaining liquid between 1'-1' and 2-2 being unaffected.

Invoking the principle of conservation of mass, the following continuity equation applies.

Fluid mass within the region 1-1 and 1'-1' = Fluid mass within the region 2-2 and 2'-2'

$$m = \rho A_1 ds_1 = \rho A_2 ds_2$$

Work done by the pressure at 1-1 in moving liquid to 1'-1'

$w_1 = \text{Force} \times \text{displacement}$

$$\Rightarrow w_1 = P_1 A_1 ds_1$$

Work done by the pressure at 2-2 in ~~the~~ moving liquid to 2'-2'

$$w_2 = -P_2 A_2 ds_2 \quad (\text{-ve sign indicates flow in reverse dir})$$

Total work done by the pressure

$$\Rightarrow w = P_1 A_1 ds_1 - P_2 A_2 ds_2$$

$$= A_1 ds_1 (P_1 - P_2) \quad (\because A_1 ds_1 = A_2 ds_2)$$

$$= \frac{m}{\rho} (P_1 - P_2) \quad (\because \frac{m}{\rho} = A \times ds)$$

Loss of potential energy = $mg(z_1 - z_2)$

Gain of kinetic energy = $\frac{m}{2}(v_2^2 - v_1^2)$

From principle of conservation of energy,

Loss of potential energy + work done by the pressure

= Gain in kinetic energy

$$\Rightarrow mg(z_1 - z_2) + \frac{m}{\rho} (P_1 - P_2) = \frac{m}{2} (v_2^2 - v_1^2)$$

Dividing the eqn through with mg

$$\Rightarrow (z_1 - z_2) + \frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

By rearranging the equation, we get

$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\Rightarrow \frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

⊛ Limitation of Bernoulli's theorem :-

1) The velocity of fluid particle in the centre of a pipe is maximum and gradually decrease towards the walls of the pipe due to friction, so only the mean velocity of the fluid should be taken into account because the velocity of fluid particles is not uniform.

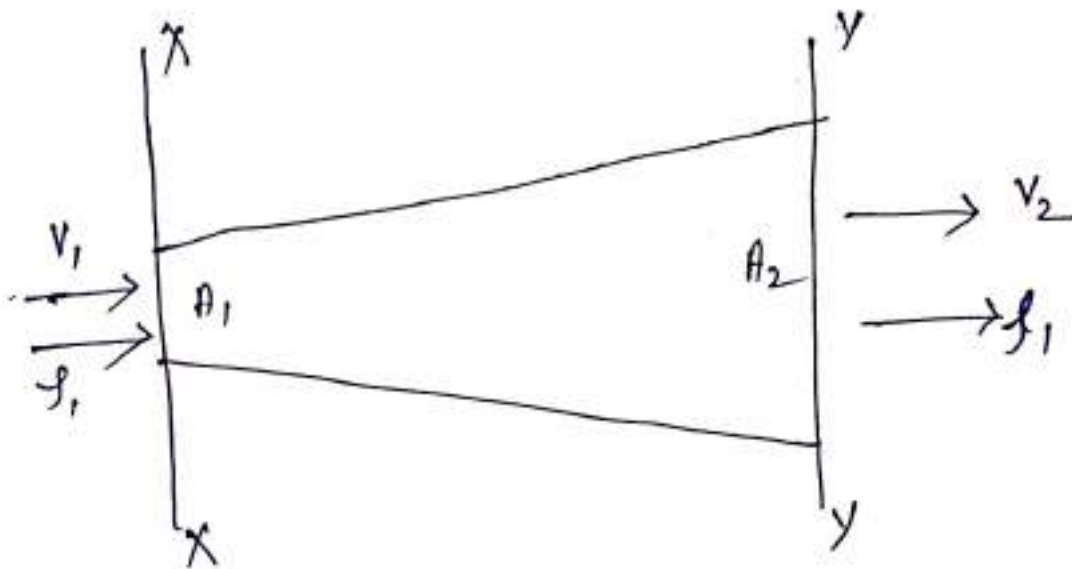
2) There are always some external forces acting on the fluid, which affect the flow of fluid, so neglect all such external forces.

3) In turbulent flow some kinetic energy is converted into heat energy and in a viscous flow some energy is lost due to shear forces, so all such losses should be neglected.

∴ If the liquid is flowing through curved path, the energy due to centrifugal forces should also be taken into account.

continuity equation

consider a pipe with two cross-section



Let, A_1 - Area of cross section at x-x

v_1 = velocity of ~~cross section~~ fluid at x-x

ρ_1 = Density of fluid at x-x

Similarly A_2, v_2, ρ_2 be the correspond values at y-y.

total mass of fluid passing through (x-x) inlet = $A_1 \times v_1 \times \rho_1$

total mass of fluid passing through (y-y) outlet = $A_2 \times v_2 \times \rho_2$

Law of mass conservation - for compressible and incompressible fluid, $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$.

$$\Rightarrow A_1 v_1 = A_2 v_2 \quad (\text{m}^3/\text{s})$$

Assumptions

While deriving the Bernoulli's equation it is assumed that

1) Flow is steady

2) irrotational

3) incompressible

4) ideal (viscosity = 0)

equation:-

$$Pv + mgz + \frac{1}{2}mv^2 = \text{constant}$$

$$\Rightarrow P \frac{m}{\rho} + mgz + \frac{1}{2}mv^2 = C$$

$$\Rightarrow \frac{P}{\rho} + gz + \frac{v^2}{2} = C \quad (\text{divide } m, \text{ constant})$$

$$\Rightarrow \frac{P}{\rho g} + z + \frac{v^2}{2g} = C \quad (\text{divide } g, \text{ constant})$$

$$\Rightarrow \frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$$

* Mass conservation

continuity equation:-

For an incompressible fluid flow the product of area and velocity remains constant. This is based on the principle of conservation of mass.

$$m/t = \text{constant}$$

$$\Rightarrow v \rho / t = \text{constant} \quad (\because \rho = \frac{m}{V})$$

$$\Rightarrow \frac{\text{Area} \times L \times \rho}{t} = \text{constant} \quad (\because V = A \times L)$$

$$\Rightarrow \frac{A \times l}{t} = \text{constant} \quad (\text{for incompressible } \rho \text{ constant})$$

$$\Rightarrow AV = \text{constant}$$

Rate of flow / discharge :-

It is defined as the volume of fluid or quantity of fluid flowing per second through a section.

It is denoted by 'Q'.

$$Q = \frac{V}{t} \text{ m}^3/\text{s}$$

$$\Rightarrow Q = AV$$

① The diameter of a pipe at two different section are respectively 10 cm. and 15 cm. Find the discharge through the pipe and the velocity at section 2. If the velocity of water at section 1 is 5 m/s.

Ans Given data,

$$d_1 = 10 \text{ cm} = 0.1 \text{ m}$$

$$v_1 = 5 \text{ m/s}$$

$$d_2 = 15 \text{ cm} = 0.15 \text{ m}$$

$$v_2 = ?$$

$$Q = ?$$

$$A_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0.1)^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$$A_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0.15)^2 = 0.01 \text{ m}^2$$

$$Q = A_1 v_1$$

$$= 7.85 \times 10^{-3} \times 5$$

$$= 0.039 \text{ m}^3/\text{s}$$

$$\therefore Q = A_2 v_2$$

$$\Rightarrow 0.039 = 0.01 \times v_2$$

$$\Rightarrow v_2 = \frac{0.039}{0.01} = 3.9 \text{ m/s}$$

② Water is blowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 . If the mean velocity is 2 m/s . Find the total energy head at a point which is 5 m above datum line.

Ans: Given data,

$$d = 5 \text{ cm} = 0.05 \text{ m}$$

$$P = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$$

$$V = 2 \text{ m/s}$$

$$z = 5 \text{ m}$$

[Potential (unit) = m]

$$T = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

(Potential) (velocity) (datum)

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{(2)^2}{2 \times 9.81} + 5$$

$$= 35.20 \text{ m}$$

③ Water is blowing through a pipe having diameters 20 cm and 10 cm at a section 1 and 2 respectively. The rate of flow through the pipe is 35 l/s . The section 1 is 6 m above the datum and section 2 is 4 m above the datum. If the pressure at section 1 is 39.24 N/cm^2 . Find the pressure at section 2.

Ans: Given data,

$$d_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$d_2 = 10 \text{ cm} = 0.1 \text{ m}$$

$$Q = 35 \text{ l/sec} = 35 \times 10^{-3} \text{ m}^3/\text{s}$$

$$z_1 = 6 \text{ m}$$

$$z_2 = 4 \text{ m}$$

[$1 \text{ m}^3 = 1000 \text{ l}$]

$$P_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$P_2 = ?$$

we know

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0.2)^2 = 0.03 \text{ m}^2$$

$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0.1)^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$$v_1 = \frac{Q}{a_1} = \frac{35 \times 10^{-3}}{0.03} = 1.16 \text{ m/s}$$

$$v_2 = \frac{Q}{a_2} = \frac{35 \times 10^{-3}}{7.85 \times 10^{-3}} = 4.45 \text{ m/s}$$

$$\therefore \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\Rightarrow \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.16)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.45)^2}{2 \times 9.81} + 4$$

$$\Rightarrow 40 + 0.06 + 6 = \frac{P_2}{9810} + 1.00 + 4$$

$$\Rightarrow P_2 = (46.6 - 5) \times 9810 = 4.6 \times 9810$$

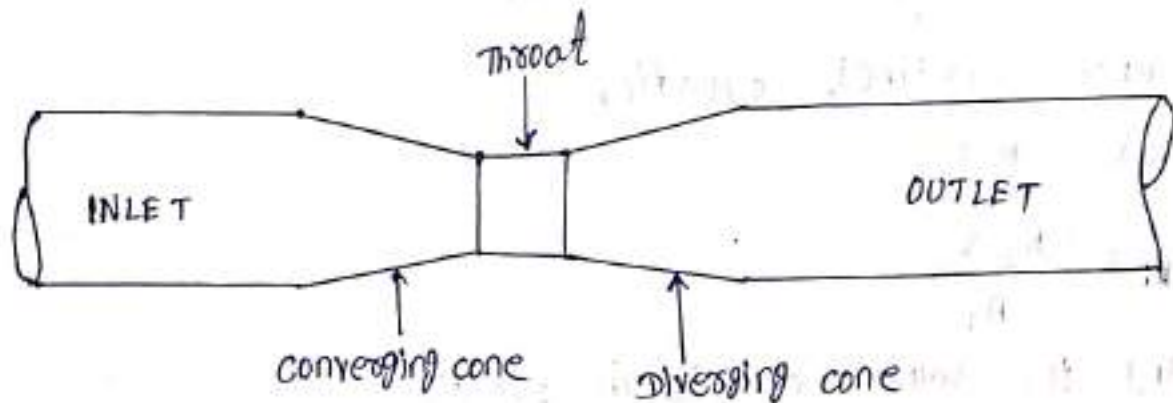
$$\Rightarrow P_2 = 408096 = 4.08 \times 10^5 \text{ N/m}^2$$

$$\frac{P}{\rho g} = 9$$

$$= 9 \text{ m}$$

Venturimeter

It is a device for measuring rate of flow in a pipe line. Its theoretical analysis is based on Bernoulli's equation and continuity equation.



- cross sectional
 A_1 = area of inlet section
 V_1 = velocity of inlet section
 A_2 = cross sectional area of throat
 V_2 = velocity of throat section

NOTE

increase in area - decrease in velocity
 decrease in velocity - decrease in kinetic energy
 decrease in kinetic energy - increase in pressure energy } motion fluid
 increase in area - decrease in pressure } static fluid

Applying Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad (\because Z_1 = Z_2)$$

$$\Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\therefore \Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h$$

$$\Rightarrow h = \frac{V_2^2 - V_1^2}{2g} \quad \text{----- (i)}$$

We know continuity equation

$$Q = A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_1 = \frac{A_2 V_2}{A_1}$$

Putting the value of V_1 in eqn (i)

$$\Rightarrow h = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow h = \frac{V_2^2 - \left(\frac{A_2 \times V_2}{A_1}\right)^2}{2g}$$

$$\Rightarrow h = \frac{V_2^2}{2g} \left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$\Rightarrow h = \frac{V_2^2}{2g} \left(\frac{A_1^2 - A_2^2}{A_1^2}\right)$$

$$\Rightarrow V_2^2 = 2gh \frac{A_1^2}{A_1^2 - A_2^2}$$

$$\Rightarrow V_2 = \sqrt{2gh} \frac{A_1}{\sqrt{A_1^2 - A_2^2}}$$

$$\therefore Q = A_2 V_2$$

$$\Rightarrow A_2 \sqrt{2gh} \frac{A_1}{\sqrt{A_1^2 - A_2^2}} = \sqrt{2gh} \frac{A_1 \times A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$C_d = \frac{Q_{act}}{Q_{th}}$$

$$Q_{act} = C_d \times Q_{th}$$

$$= C_d \times a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

Manometer reading (x)

Relation between h and x

$$h = x \left(\frac{\rho_{mh}}{\rho_s} - 1 \right) \quad \text{--- (i)}$$

$$h = x \left(1 - \frac{\rho_{ml}}{\rho_s} \right) \quad \text{--- (ii)}$$

where, h = system fluid pressure difference

x = Manometer head difference

ρ_{mh} = Manometric fluid density ($\rho_{mh} > \rho_s$)

ρ_{ml} = Manometric fluid density ($\rho_{ml} < \rho_s$)

ρ_s = system fluid density.

- ① A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the rate of flow of water. If the differential manometer connected between the inlet and throat gives a reading of 20 cm of mercury then calculate the actual rate of flow taking the coefficient of discharge as 0.98.

Ans: Given data.

system fluid - water

Manometric fluid - Mercury

$$C_d = 0.98$$

$$d_1 = 30 \text{ cm} = 0.3 \text{ m}$$

$$d_2 = 15 \text{ cm} = 0.15 \text{ m}$$

$$x = 20 \text{ cm} = 0.2 \text{ m}$$

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0.2)^2 = 0.070 \text{ m}^2$$

$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0.15)^2 = 0.017 \text{ m}^2$$

$$h = \chi \left(\frac{f_{mh}}{f_s} - 1 \right)$$

$$= 0.2 \left(\frac{13600}{1000} - 1 \right) = 2.52 \text{ m}$$

$$Q_{act} = C_d a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= 0.98 \times 0.070 \times 0.017 \sqrt{\frac{2 \times 9.81 \times 2.52}{(0.070)^2 - (0.017)^2}}$$

$$= 0.120 \text{ m}^3/\text{s}$$

- ② An oil of specific gravity 0.8 is flowing through a venturimeter of cross section $20 \text{ cm} \times 10 \text{ cm}$. The mercury differential manometer gives a reading of 25 cm . Calculate the discharge of oil taking C_d as 0.98.

Ans: Given data,

system fluid - oil

Manometric fluid - mercury

SP. gr. oil = 0.8

$\rho_{oil} = 800 \text{ kg/m}^3$

$d_1 = 20 \text{ cm} = 0.2 \text{ m}$

$d_2 = 10 \text{ cm} = 0.1 \text{ m}$

$C_d = 0.98$

$\chi = 25 \text{ cm} = 0.25 \text{ m}$

$$a_1 = \frac{\pi}{4} \times (0.2)^2 = 0.031$$

$$a_2 = \frac{\pi}{4} \times (0.1)^2 = 7.853 \times 10^{-3}$$

$$h = x \left(\frac{f_{ml}}{f_s} - 1 \right)$$

$$= 0.25 \left(\frac{13600}{800} - 1 \right) = 4 \text{ m.}$$

$$Q_{act} = C_d a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= 0.98 \times 0.031 \times 7.853 \times 10^{-3} \sqrt{\frac{2 \times 9.81 \times 4}{(0.031)^2 - 6(7.853 \times 10^{-3})^2}}$$

$$= 0.07 \text{ m}^3/\text{s.}$$

26.04.23

- ② A horizontal venturimeter with inlet diameter 20 cm and the throat diameter 10 cm is used to measure the flow of water. The pressure at the inlet is 17.658 N/cm^2 and the vacuum pressure at the throat is 30 cm of Hg. Find the discharge of water through venturimeter taking coefficient of discharge as 0.98.

Ans: $d_1 = 20 \text{ cm} = 0.2 \text{ m}$

$d_2 = 10 \text{ cm} = 0.1 \text{ m}$

system fluid - water

manometric fluid - mercury

$P_2 = -30 \text{ cm Hg} = \dots = -0.3 \text{ m of Hg}$

$P_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

$C_d = 0.98$

$$a_1 = \frac{\pi}{4} \times (d_1)^2$$

$$= \frac{\pi}{4} \times (0.2)^2 = 0.031$$

$$a_2 = \frac{\pi}{4} \times (d_2)^2$$

$$= \frac{\pi}{4} \times (0.1)^2 = 7.853 \times 10^{-3}$$

$$h = \text{pressure head at inlet} - \text{pressure head at throat}$$

$$= \frac{17.658 \times 10^4}{1000 \times 9.81} - \frac{0.3 \times 13600}{1000}$$

$$= 18 - (-4.08)$$

$$= 22.08 \text{ m}$$

$$Q_{\text{act}} = C_d a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= 0.98 \times 0.031 \times 7.853 \times 10^{-3} \sqrt{\frac{2 \times 9.81 \times 22.08}{(0.031)^2 - (7.853 \times 10^{-3})^2}}$$

$$= 0.16 \text{ m}^3/\text{s}$$

$$= 16 \text{ cm}^3/\text{s} \cdot (\text{Ans})$$

Pitot Tube

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. This works on the principle that the kinetic energy of the fluid is converted to the pressure energy and hence by applying the Bernoulli's equation we can get the expression for velocity.

Stagnation properties:-

When a fluid is flowing and at a point its the velocity becomes zero (resultant) then the values of pressure, temperature and density at that point are called stagnation properties. This point is known as stagnation point. So the different parameters are named as:- stagnation pressure
stagnation temperature
stagnation density respectively.

Pitot tube is a glass tube bent at right angles. Basically the liquid rises up in the tube due to conversion of kinetic energy into pressure energy.

By applying Bernoulli's equation we can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad (\because z_1 = z_2)$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} \quad (\because V_2 = 0 \text{ . stagnation point})$$

$$\Rightarrow \frac{V_1^2}{2g} = \frac{P_2}{\rho g} - \frac{P_1}{\rho g}$$

$$\Rightarrow \frac{V_1^2}{2g} = h$$

$$\Rightarrow V_1 = \sqrt{2gh}$$

$$\Rightarrow V_1 = \sqrt{2gh}$$

$$\Rightarrow (V_1)_{act} = C_v \sqrt{2gh}$$

- ① A pitot static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6m. and static pressure head is 5m. Calculate the velocity of flow taking the C_v as 0.98.

Ans: Given data,

$$\text{Stagnation pressure head } (h_2) = 6\text{m}$$

$$\text{static pressure head } (h_1) = 5\text{m}$$

$$h = h_2 - h_1$$

$$= 6 - 5 = 1\text{m.}$$

$$C_v = 0.98$$

$$V_{th} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1} = 4.42$$

$$V_{act} = C_v \sqrt{2gh} = 0.98 \sqrt{2 \times 9.81 \times 1} = 4.34 \text{ m/s. (Ans)}$$

- ② Find the stagnation pressure head at a point if the static pressure head is 9m. and the velocity at that point is 4.5 m/s.

Ans: Given data

$$h_2 = ?$$

$$h_1 = 9\text{m}$$

$$V_1 = 4.5 \text{ m/s}$$

$$V_1 = \sqrt{2gh}$$

$$\Rightarrow V_1 = \sqrt{2 \times 9.81 \times h}$$

$$\Rightarrow \sqrt{(4.5)^2} = \sqrt{2 \times 9.81 \times h}$$

$$\Rightarrow h = \frac{(4.5)^2}{2 \times 9.81} = 1.03\text{m}$$

We know, $h = h_2 - h_1$

$$\Rightarrow 1.03 = h_2 - 9$$

$$\Rightarrow h_2 = 1.03 + 9 = 10.03 \text{ m} \quad (\text{Ans})$$

09.05.23

③ A Pitot tube placed in a sub-marine is connected to a U-tube manometer containing mercury. The difference of mercury level is ~~is~~ 170 mm. Find the speed of the submarine.

Ans: Given data,

$$\rho_{\text{system}} = 1000 \text{ kg/m}^3$$

$$\rho_{\text{manometric}} = 13600 \text{ kg/m}^3$$

$$x = 170 \text{ mm} = 0.17 \text{ m}$$

$$V = ?$$

$$h = x \left(\frac{\rho_m}{\rho_s} - 1 \right)$$

$$= 0.17 \left(\frac{13600}{1000} - 1 \right)$$

$$= 2.142 \text{ m}$$

$$V = C_v \sqrt{2gh}$$

$$\Rightarrow V = 1 \times \sqrt{2 \times 9.81 \times 2.142}$$

$$= 6.48 \text{ m/s}$$

④ Define impact of jet.

Impact of jet means the force exerted by the jet on a plate which may be stationary or moving. This force is calculated by Newton's second law of motion.

Orifice

Orifice is a small opening generally placed at one side or at the bottom of the tank through which the fluid is flowing.

Classification of orifice :-

- ① Depending on the cross-sectional area the orifices are classified as :-
 - (i) circular
 - (ii) rectangular
 - (iii) triangular
 - (iv) square
- ② on the basis of discharge nature the orifices may be classified as :-
 - (i) free discharging
 - (ii) drowned / submerged
- ③ Based on the size of the orifice it can be classified as :-
 - (i) small orifice (head of the liquid is less than 5 times of the depth of orifice)
 - (ii) Large orifice (head of the liquid is more than 5 times of the depth of orifice)

Discharge coefficient :-

① Coefficient of discharge :- $(0.61 - 0.65)$, $C_d = 0.62$

It is the ratio of actual discharge from an orifice to that of the theoretical discharge from the orifice.

$$C_d = \frac{Q_{act}}{Q_{th}}$$

(ii) coefficient of contraction :- (0.61 - 0.69), $C_c = 0.64$

It is defined as the ratio of the area of the jet at vena-contracta to that of the area of the orifice.

$$C_c = \frac{\text{Area of venacontracta}}{\text{Area of orifice}} = \frac{a_c}{a_0}$$

(iii) coefficient of velocity :- (0.95 - 0.99), $C_v = 0.98$

It is the ratio between the actual velocity of a jet of liquid at vena-contracta to that of the theoretical velocity of the jet.

$$C_v = \frac{V_{act}}{\sqrt{2gH}}$$

Relation between C_c , C_d and C_v .

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{a_c \times V_c}{a_0 \times V_{th}} \\ = \left(\frac{a_c}{a_0} \right) \times \left(\frac{V_c}{V_{th}} \right)$$

$$C_d = C_c \times C_v$$

Flow through an orifice

Let us consider a tank fitted with a circular orifice on one of its sides. The fluid goes on flowing through this orifice and we can observe that the area of the jet goes on decreasing. So the section at which the area is minimum is known as vena-contracta.

considers two points 1 and 2 as shown in fig. where point 1 is the point of observation inside the tank and point 2 is at the venacontracta (level of 1 and 2 is same). Applying Bernoulli's theorem we can write

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

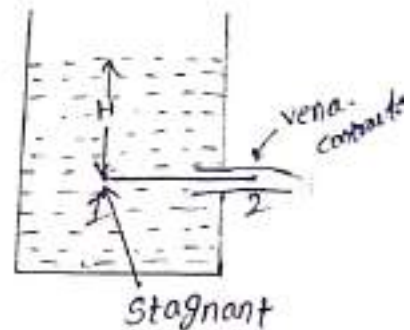
$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad (\because z_1 = z_2)$$

$$\Rightarrow \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad (\because v_1 = 0)$$

$$\Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{v_2^2}{2g}$$

$$\Rightarrow H = \frac{v_2^2}{2g}$$

$$\Rightarrow v_2 = \sqrt{2gH}$$



① The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at venacontracta. Take $C_d = 0.6$ and $C_v = 0.98$.

Ans: Given data,

$$C_d = 0.6$$

$$C_v = 0.98$$

$$d_o = 40 \text{ mm}$$

$$H = 10 \text{ m}$$

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14$$

$$C_v = \frac{V_{\text{vena contracta}}}{V_{th}}$$

$$\begin{aligned} \Rightarrow V_{\text{vena contracta}} &= C_v \times V_{th} \\ &= 0.98 \times 14 \\ &= 13.73 \text{ m/s (Ans)} \end{aligned}$$

$$\begin{aligned} A_0 &= \frac{\pi}{4} \times d^2 \\ &= \frac{\pi}{4} \times (0.04)^2 = 1.25 \times 10^{-3} \end{aligned}$$

$$\begin{aligned} Q_{th} &= A_0 \times V_{th} \\ &= 1.25 \times 10^{-3} \times 14 = 0.0175 \end{aligned}$$

$$\begin{aligned} Q_{act} &= C_d \times Q_{th} \\ &= 0.6 \times 0.0175 = 0.0105 \text{ (Ans)} \end{aligned}$$

② The head of water over the centre of an orifice of diameter 20mm is 1m. The actual discharge through the orifice is 0.85 l/s. Find the coefficient of discharge. If the C_v is 0.95 then calculate the C_c .

Ans: $d_0 = 20 \text{ mm} = 0.02 \text{ m}$

$H = 1 \text{ m}$

$Q_{act} = 0.85 \text{ l/s} = 0.85 \times 10^{-3} \text{ m}^3/\text{s}$

$C_d = ? \frac{Q_{act}}{Q_{th}}$

$C_v = 0.95$

$C_c = ?$

$Q_{th} = A_0 \times V_{th}$

$\therefore A_0 = \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (0.02)^2 = 3.14 \times 10^{-4}$

$V_{th} = \sqrt{2gh}$

$= \sqrt{2 \times 9.81 \times 1} = 4.42$

$$Q_{th} = Q_b \times V_{th} \\ = 3.14 \times 10^{-4} \times 4.42 = 1.38 \times 10^{-3}$$

$$C_d = \frac{Q_{act}}{Q_{th}} \\ = \frac{0.85 \times 10^{-3}}{1.38 \times 10^{-3}} = 0.61 \text{ (ANS)}$$

$$C_d = C_c \times C_v$$

$$\Rightarrow C_c = \frac{C_d}{C_v} = \frac{0.61}{0.95} = 0.64 \text{ (ANS)}$$

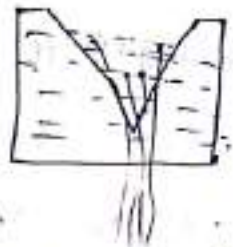
UNIT-5

Notches :-

A notch is a device used for measuring the rate of flow of liquid through a small channel or a tank.

→ The bottom edge, over which the liquid flows, is known as sill or crest of the notch and the sheet of liquid flowing over a notch (or a weir) is known as nappe or vein.

→ A notch is usually made of a metallic plate and is used to measure discharge of liquids.



Weir :-

A weir is a concrete or masonry structure, placed in an open channel over which the flow occurs.

→ It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel.

*) The notch is of small size, while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonry structure.

Classification of notches

*) According to the shape of opening

- Rectangular notch
- Triangular notch
- Trapezoidal notch
- Stepped notch

- *) According to the effect of the sides on the nappe
 - notch with end contraction.
 - notch without end contraction or suppressed notch.

Classification of weirs

- *) According to the shape of opening
 - Rectangular weirs
 - Triangular weirs
 - Trapezoidal weirs

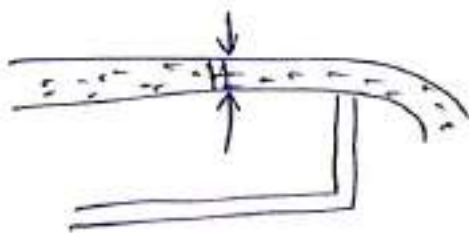
- *) According to the shape of crest
 - Sharp-crested
 - Broad-crested
 - Narrow-crested
 - Ogee-shaped

- *) According to the effect of sides on the emerging nappe
 - Weirs with end contraction
 - Weirs without end contraction

- Weirs with end contraction
- Weirs without end contraction

Discharge over rectangular notch and weirs

The expression for discharge over rectangular notch or weir is the same.



consider a rectangular notch provided in a channel carrying water as shown.

Let H = head of water over the crest.

L = Length of notch.

For finding the discharge of water flowing over notch, consider a elemental horizontal strip length L , depth dh from the free surface of water.

Area of strip = $L \times dh$

Theoretical velocity = $\sqrt{2gh}$

discharge (dce) through strip = $C_d \times \text{Area of strip} \times \text{theoretical velocity}$
 $= C_d \times L \times dh \times \sqrt{2gh}$

total discharge $\Rightarrow Q = \int_0^H C_d \cdot L \cdot \sqrt{2g} h \cdot dh$

$$= C_d \cdot L \cdot \sqrt{2g} \int_0^H h^{1/2} dh$$

$$= C_d \cdot L \cdot \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H$$

$$= C_d \cdot L \cdot \sqrt{2g} \left[\frac{h^{3/2}}{\frac{3}{2}} \right]_0^H$$

$$= \frac{2}{3} C_d \cdot L \cdot \sqrt{2g} [H^{3/2}]$$

① Calculate the discharge over a rectangular notch whose length is 2m and head over the notch is 0.3m. Take $C_d = 0.623$.

Ans: Given data,

$$L = 2\text{m}$$

$$H = 0.3\text{m}$$

$$C_d = 0.623$$

$$Q = ?$$

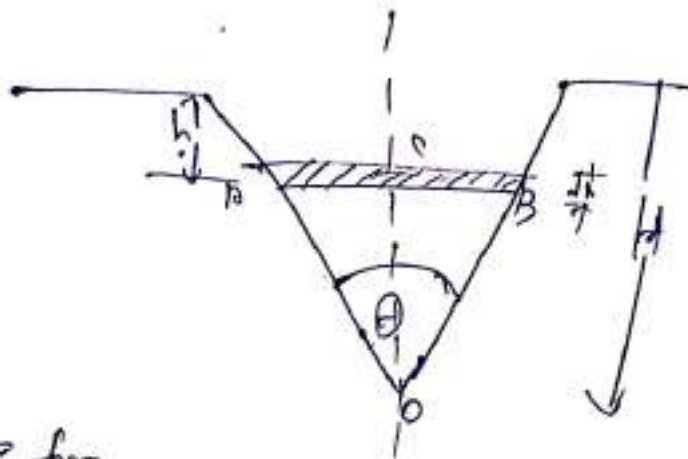
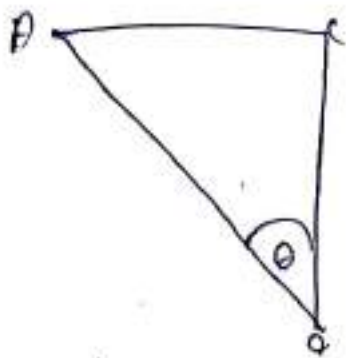
$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2}$$

$$= \frac{2}{3} \times 0.623 \times 2 \times \sqrt{2 \times 9.81} \times (0.3)^{3/2}$$

$$= 0.598 \text{ m}^3/\text{sec} \quad (\text{Ans})$$

④ Discharge of triangular notch and weir

$$Q = \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} [H]^{5/2}, \text{ m}^3/\text{sec}$$



Let H & B distance \propto from
 O or origin of notch.

Consider a horizontal strip of water thickness dh , at a depth h

$$\tan \frac{\theta}{2} = \frac{AC}{OC} = \frac{BC}{(H-h)}$$

Area $AC = (H-h) \tan \frac{\theta}{2}$
 width of strip $\cdot AB = 2AC$
 $= 2 \times$

Area of strip $= 2 \times (H-h) \tan \frac{\theta}{2} \times dh$

theoretical velocity of water $= \sqrt{2gh}$

discharge $\cdot dQ \propto 2(H-h) \tan \frac{\theta}{2} \times dh \times \sqrt{2gh}$

$= 2 C_d (H-h) \tan \frac{\theta}{2} \sqrt{2gh} dh$

total disch

$Q = \int_0^H$

$= 2 C_d \tan \frac{\theta}{2} \sqrt{2g} \int_0^H (H-h) h dh$

$= \int_0^H (H \cdot h - h^2) dh$

$= \left[\frac{H \cdot h^2}{2} - \frac{h^3}{3} \right]_0^H$

$$\propto 2cd \tan^2 \frac{\theta}{2} \sqrt{2g} \left(\frac{2}{3} \omega \cdot 10^2 - \frac{2}{5} 10^5 \right)$$

$$\propto 2cd \tan^2 \frac{\theta}{2} \sqrt{2g} \times \left(\frac{2}{3} 10^2 - \frac{2}{5} 10^5 \right)$$

$$\propto 2cd \tan^2 \frac{\theta}{2} \sqrt{2g} \left(\frac{4}{15} 10^2 \right)$$

$$\propto \frac{0}{15} \propto cd \tan^2 \frac{\theta}{2} \sqrt{2g} \propto 10^2$$

UNIT - 6FLOW THROUGH PIPELOSS OF ENERGY IN PIPES

The fluid experiences some resistance while flowing through a pipe due to which the energy of the fluid is lost. The energy losses can be broadly classified into two categories. (i) Major energy loss (energy loss due to friction) (ii) Minor energy loss (due to sudden expansion or sudden contraction of pipe or bend or obstruction in pipe or due to pipe fittings)

LOSS OF ENERGY OR ENERGY HEAD DUE TO FRICTION

There are two ways to determine the energy loss due to friction.

(i) Darcy-Weisbach formula

using this formula we can write loss of head due to friction

$$h_f = \frac{4fLV^2}{d2g}$$

where, L = length of pipe

V = ^{mean} velocity of flow

d = diameter of pipe

f = coefficient of friction

$$= \frac{16}{Re} \quad \text{for } Re < 2000$$

or

$$= \frac{0.0719}{Re^{1/4}}$$

① Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m through which water is flowing at a velocity of 3 m/s. The kinematic viscosity for water is equal to 0.01 stoke.

Ans: Given data,

$$d = 300 \text{ mm} = 0.3 \text{ m}$$

$$L = 50 \text{ m}$$

$$V = 3 \text{ m/s}$$

$$\nu_w = 0.01 \text{ stoke} = 1 \times 10^{-6} \text{ m}^2/\text{sec}$$

$h_f = ?$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.3}{1 \times 10^{-6}} = 900000$$

$$f = \frac{0.0719}{Re^{1/4}} = \frac{0.0719}{(900000)^{1/4}} = 2.33 \times 10^{-3}$$

$$\text{Loss } h_f = \frac{4fLV^2}{d2g} = \frac{4 \times 2.33 \times 10^{-3} \times 50 \times (3)^2}{0.3 \times 2 \times 9.81} = 0.71 \text{ m}$$

(ii) Chezy's Formula :-

The loss of head due to friction can also be determined by this formula which is given as

$$\Rightarrow V = C \sqrt{mi}$$

where, V = mean velocity of fluid flow

m = hydraulic mean depth, which is the ratio of area of flow to that of the perimeter. $= \frac{A}{P}$

i = It is the head of loss per unit length of pipe

$$i = \frac{h_f}{L}$$

C = Chezy's constant

Q Find the head lost due to friction in a pipe of diameter 300mm and length 50m through which water is flowing at a velocity of 3 m/s. Take Chezy's constant 60.

Ans) Given data,

$$d = 300 \text{ mm} = 0.3 \text{ m}$$

$$L = 50 \text{ m}$$

$$V = 3 \text{ m/s}$$

$$C = 60$$

$$h_f = ?$$

$$V = C \sqrt{mi}$$

$$\Rightarrow V = 60 \sqrt{0.075 \times \frac{h_f}{L}}$$

$$\Rightarrow V = 60 \sqrt{0.075 \times \frac{h_f}{50}}$$

$$\Rightarrow 3^2 = 60^2 \times 0.075 \times \frac{h_f}{50}$$

$$\Rightarrow h_f = \frac{3^2 \times 50}{60^2 \times 0.075} = 1.66 \text{ m}$$

$$m = \frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4}$$

$$= \frac{0.3}{4} = 0.075$$

Hydraulic gradient line :-

It is defined as the line which gives the sum of pressure head and datum head of a fluid flowing in a pipe. It is briefly written as 'HGL'.

$$\text{HGL} = \frac{P}{\omega} + z$$

where, $\frac{P}{\omega}$ = pressure head

z = datum head

Total energy line :-

It is briefly written as TEL. which gives the sum of pressure head, datum head and kinetic head of a fluid flowing in a pipe.

$$\text{TEL} = \frac{P}{\omega} + \frac{v^2}{2g} + z$$

Pipe :-

- Fluids are transported most often in pipes.
- pipe is a heavy walled, large in diameter.
- Pipes comes in moderate length of 20-40 ft.
- pipe walls are usually rough.
- pipes are joined by screw and flanges or welding fitting.

UNIT-7

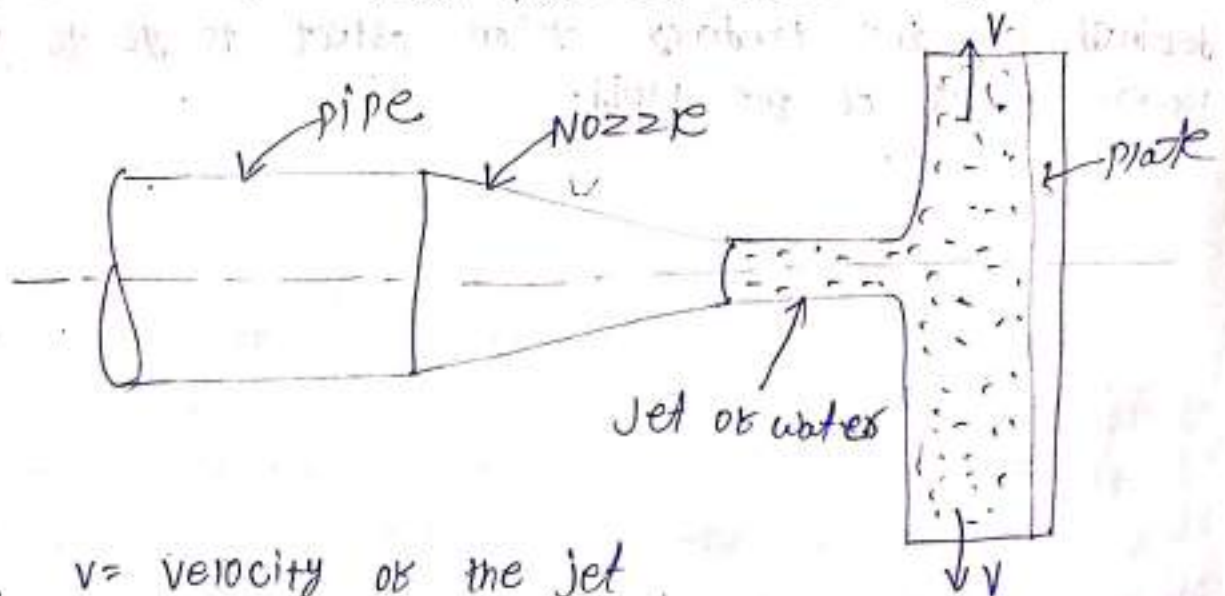
Impact of Jet :-

Impact of jet means the force exerted by the jet on a plate which may be stationary or moving. This force is calculated by Newton's second law.

- (*) Newton's second law states that the acceleration of an object is dependent upon two variables - the net force acting upon the object and the mass of the object.

Force exerted by the jet on a fixed vertical flat plate

Consider a jet of water coming out from the nozzle strikes a flat vertical plate as shown in figure 1.



Let, v = velocity of the jet

d = diameter of the jet

a = area of cross-section of the jet

$$= \frac{\pi}{4} \times d^2$$

- The jet after striking the plate, will move along the plate. But the plate is at right angle to the jet.
- Hence, the jet after striking will get deflected through 90° .

- Therefore, the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet, $F_x = \rho a v^2$

F_x = Rate of change of momentum by the jet on the plate in the direction of jet, ~~$F_x = \rho a v^2$~~

F_x = Rate of change of momentum in the direction of force

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity}) - (\text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} (\text{Initial velocity} - \text{Final velocity})$$

$$= \frac{\text{Mass}}{\text{sec}} \times (\text{velocity of the jet before striking} - \text{Final velocity of jet after striking})$$

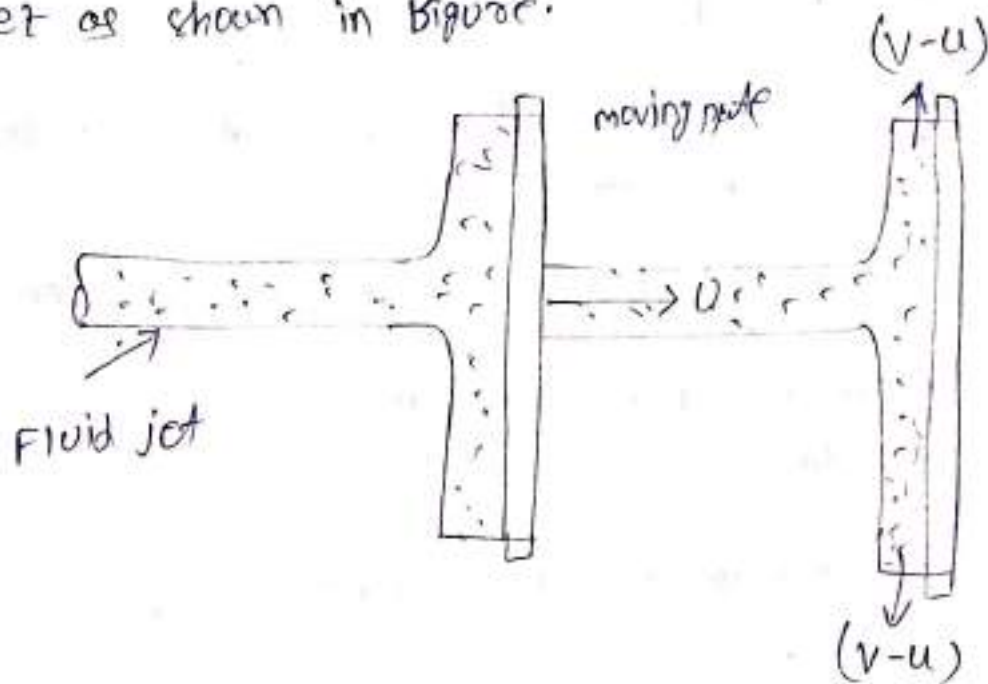
$$= \rho a v (v - 0) \quad (\because \text{Mass/sec} = \rho \times a v)$$

$$\Rightarrow F_x = \rho a v^2$$

Unit - N

Force exerted by a jet on moving vertical flat plate

Consider, a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet as shown in figure.



Let, v = velocity of the jet

a = cross sectional area of jet

u = velocity of the flat plate

In this case, the jet does not strike the plate with a velocity v , but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence,

Relative velocity of the jet with respect to plate = $(v-u)$

∴ Mass of water striking the plate per sec,

$$= \rho \times \text{Area of jet} \times \text{velocity of jet}$$

$$= \rho a \times (v-u)$$

∴ Force exerted by the jet on the moving plate in the direction of the jet,

$$F_x = \text{Mass of water striking per sec} \times [\text{initial velocity} - \text{final velocity}]$$

$$= \rho a (v-u) [(v-u) - 0] \quad (\because \text{Final velocity} = 0)$$

$$= \rho a (v-u)^2.$$

In this case, the work will be done by the jet on the plate, as the plate is moving

∴ work done per second by the jet on the plate

$$W = \text{Force} \times \frac{\text{distance moved by the plate in the direction of jet}}{\text{time}}$$

$$= F_x \times U$$

$$= \rho a (v-u)^2 \times U.$$

